Math 8174: Homework 1

Due January 21, 2009

1. Consider the symmetric group S_4 and the dihedral group D_8. For each group G

   (a) Give examples of two nonequivalent and nontrivial representations ρ and τ (be sure to show they are not equivalent),

   (b) Construct the corresponding G-modules V_ρ and V_τ,

   (c) Decide whether the modules are reducible,

   (d) Change bases in the module V_ρ and give the new corresponding representation $\rho' : G \to GL_n(\mathbb{C})$.

2. Show that if $\rho : G \to GL(V)$ is a degree one representation, then $G/ker(\rho)$ is an abelian group.

3. Let $\mathrm{GL}_2(\mathbb{F}_q)$ be the general linear group of rank 2 with entries in the field $\mathbb{F}_q$ with q elements. Consider the subalgebra of $\mathbb{C}\mathrm{GL}_2(\mathbb{F}_q)$ given by

$$\mathcal{H}_2(q) = e_B \mathbb{C}\mathrm{GL}_2(\mathbb{F}_q) e_B, \qquad \text{where} \quad e_B = \frac{1}{q} \sum_{\substack{r,s \in \mathbb{F}_q^\times \\ t \in \mathbb{F}_q}} \begin{pmatrix} r & t \\ 0 & s \end{pmatrix}.$$

   (This is the Iwahori-Hecke algebra $\mathcal{H}_2(q)$).

   (a) Find a basis for $\mathcal{H}_2(q)$.

   (b) Give formulas for multiplying basis elements.

   (c) Construct a nontrivial $\mathcal{H}_2(q)$-module that is not the regular module.