

# Math 8174: Homework 6

Due: November 19, 2010

1. For  $\mathfrak{g} = \mathfrak{sp}_4(\mathbb{C})$  and  $\lambda = 2\omega_\alpha + 3\omega_\beta$ , compute

(a)  $\Lambda(L(\lambda))$ ,

(b) For each  $\mu \in \Lambda(L(\lambda))$ , bound the dimension  $\dim(L(\lambda)_\mu)$ .

2. Let  $\lambda \in P^+$ . Show that 0 is a weight of  $L(\lambda)$  if and only if  $\lambda \in Q$ .

3. If  $V$  and  $U$  are two  $\mathfrak{g}$ -modules, then  $U \otimes V$  is the  $\mathfrak{g}$ -module given by

$$x \cdot (u \otimes v) = (x \cdot u) \otimes v + u \otimes (x \cdot v).$$

(a) Show that  $\Lambda(L(\lambda) \otimes L(\mu)) = \Lambda(L(\lambda)) + \Lambda(L(\mu))$ .

(b) Show that

$$\dim((L(\lambda) \otimes L(\mu))_{\nu+\nu'}) = \sum_{\gamma+\gamma'=\nu+\nu'} \dim(L(\lambda)_\gamma) \dim(L(\mu)_{\gamma'}).$$

See page two for  $B_2$  paper.

