

Math 8174: Homework 4

Due: October 20, 2010

1. Let $R \subseteq E$ be a finite subset satisfying all axioms for a root system except

$$(R2) \text{ If } r\alpha, \alpha \in R, \text{ then } r = \pm 1.$$

Show that if $c\alpha, \alpha \in R$, then $c = \pm\frac{1}{2}, \pm 1, \pm 2$.

2. (a) State the theorem that classifies Cartan matrices of simple Lie algebras.
(b) Find the determinants of the matrices of (a).
3. Show that the Weyl group of a reducible root system factors as a direct product of Weyl groups of the irreducible pieces of the root system.
4. Prove that if a Dynkin diagram contains another, then the corresponding root systems exhibit a corresponding containment.