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# Math 8174: Homework 2

## Due: September 22, 2010

1. For $\mathfrak{g} = \mathfrak{sl}_n(\mathbb{C})$ and either $\mathfrak{g} = \mathfrak{sp}_{2n}(\mathbb{C})$ or $\mathfrak{g} = \mathfrak{o}_n(\mathbb{C})$ do the following:
   (a) Find a Cartan subalgebra $\mathfrak{h}$.
   (b) Find a basis of $\mathfrak{g}$ compatible with the Cartan decomposition of $\mathfrak{g}$ with respect to $\mathfrak{h}$ (this requires finding the decomposition).

2. Let $\mathfrak{g}$ be a simple Lie algebra.
   (a) Show that if $(\cdot,\cdot) : \mathfrak{g} \otimes \mathfrak{g} \to \mathbb{C}$ is a symmetric, nondegenerate, bilinear form such that $([x,y],z) = (x,[y,z])$, then there exists $\lambda \in \mathbb{C}$ such that $\kappa(\cdot,\cdot) = \lambda(\cdot,\cdot)$.
   Hint: Study a composition of linear transformations $\mathfrak{g} \to \mathfrak{g}^* \to \mathfrak{g}$.
   (b) Assuming that $\mathfrak{sl}_n(\mathbb{C})$ is simple show that

$$\kappa(x,y) = 2n\mathrm{tr}(xy), \qquad \text{for } x,y \in \mathfrak{sl}_n(\mathbb{C}).$$

3. Show that if $\mathfrak{g}$ is a semi-simple Lie algebra, then $[\mathfrak{g},\mathfrak{g}] = \mathfrak{g}$.

4. Show that if $\mathfrak{g}$ is solvable, then

$$\kappa(x,y) = 0, \qquad \text{for all } x,y \in \mathfrak{g}.$$