

Math 6350: Homework 4

Due: Friday, October 19

A. Evaluate the following integrals using the residue theorem.

$$(1) \int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx,$$

$$(2) \int_0^{\infty} \frac{\sqrt{x}}{1 + x^3} dx,$$

$$(3) \int_{-\infty}^{\infty} \frac{\cos(x)}{4x^2 - \pi^2} dx,$$

$$(4) \int_{-\infty}^{\infty} \frac{\cos(3x)}{e^x + e^{-x}} dx.$$

B. Find the number of roots of the following polynomials in the specified region (See Ahlfors Exercises 5.2.1 and 5.2.3).

$$(1) z^7 - 2z^5 + 6z^3 - z + 1 \text{ in } |z| < 1,$$

$$(2) z^4 + 8z^3 + 3z^2 + 8z + 3 \text{ in } \operatorname{Re}(z) > 0.$$