

Math 6140: Homework 8

1. 13.3: 4
2. 13.4: 4, 5, 6
3. Let $\mathbb{F} \subseteq \mathbb{K} \subseteq \mathbb{L}$. Suppose $\alpha \in \mathbb{L}$ is algebraic over \mathbb{F} and let $f = \min_{\alpha, \mathbb{K}}(x)$. Show that the roots in \mathbb{L} and coefficients of f are algebraic over \mathbb{F} .
4. Suppose $x^p - 1$ factors completely over a field \mathbb{F} with p prime. Show that for each $a \in \mathbb{F}$, either $x^p - a$ factors completely in $\mathbb{F}[x]$ or is irreducible in $\mathbb{F}[x]$ (Hint: note that the roots of $x^p - a$ all have the same degree).