

Math 6140: Homework 4

1. 11.4: 4

2. Let $A \in M_{n \times n}(\mathbb{F})$. For $I, J \subseteq \{1, 2, \dots, n\}$, let $A_{I,J} \in M_{|I| \times |J|}(\mathbb{F})$ be the submatrix of A obtained by taking only the rows in I and the columns in J . A matrix $A \in M_{n \times n}(\mathbb{Z})$ is totally positive, if for all $I, J \subseteq \{1, 2, \dots, n\}$ with $1 \leq |I| = |J| \leq n$, we have

$$\det(A_{I,J}) > 0.$$

- (a) Show that totally positive matrices can be similar to non-totally positive matrices.
- (b) Classify all totally positive matrices in $M_{2 \times 2}(\mathbb{Z})$ (give conditions on the coordinates that guarantee total positivity).

3. 12.1: 2, 5, 6, 9, 11, 12, 14