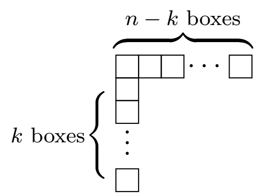
Math 4140: Homework 8

Due: March 16, 2011

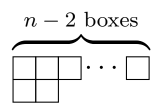
Required

1. For the following shapes, determine the dimensions of the corresponding S_n-modules.

   (a)

   (b)

2. Suppose an S_7-module V contains a vector v such that

$$v * m_1 = 0, v * m_2 = -v, v * m_2 = -2v, v * m_3 = v, v * m_4 = 2v,$$

$$v * m_5 = -3v, v * m_6 = 0, v * m_7 = -v.$$

   Identify an irreducible submodule of V containing this vector (your answer should be in terms of a specific integer partition).

3. Recall that the permutation module V of S_n has a module decomposition

$$V = \mathbb{C}\text{-span}\{v_1 + v_2 + \cdots + v_n\} \oplus \{a_1 v_1 + a_2 v_2 + \cdots + a_n v_n \mid a_1 + a_2 + \cdots + a_n = 0\}.$$

   We know that

$$\mathbb{C}\text{-span}\{v_1 + v_2 + \cdots + v_n\} \cong S_n^{\square\square\square\cdots\square}.$$

   This problem seeks to understand the other piece $W_n = \{a_1 v_1 + a_2 v_2 + \cdots + a_n v_n \mid a_1 + a_2 + \cdots + a_n = 0\}$.

   (a) Let $n = 4$. What is the dimension d of W_4?

   (b) Find d linearly independent vectors $w \in W_4$ such that

$$w * m_k = c_k w$$

   for all $1 \leq k \leq 4$ and for some $c_k \in \mathbb{C}$. That is, find d simultaneous eigenvectors for the Murphy-Jucys elements.

   (c) Use these eigenvectors to give you an isomorphism between W_4 and an irreducible module of S_4.