# Math 4140: Homework 5

Due: February 16, 2011

## Required

1. Consider the infinite group $\mathbb{Z}$ under addition.

   (a) Show that

   $$\begin{array}{rccc} \rho: & \mathbb{Z} & \longrightarrow & \mathrm{GL}_2(\mathbb{C}) \\ & m & \mapsto & \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix} \end{array}$$

   is a representation of $\mathbb{Z}$.

   (b) Let $V$ be the corresponding module. Show that the conclusion of Maschke's theorem does not apply to some nonzero proper submodule $U \subseteq V$.

   **Remark:** A module which is not irreducible, but does not satisfy the conclusion of Maschke's Theorem is called *indecomposible.*

2. (a) Classify the irreducible modules of $C_m \times C_n$ where $m, n \in \mathbb{Z}_{\geq 1}$.

   (b) Generalize (a) by using the fundamental theorem of abelian groups to classify all the irreducible modules of an arbitrary abelian group $G$.

3. Recall, that $C_4 \subseteq D_8$. Use Maschke's Theorem to decompose the regular module $\mathbb{C}D_8$ as a sum of irreducible $C_4$-modules (you should have 8 irreducible modules).

4. For $G$-modules $U$ and $V$, define

   $$\mathrm{Hom}_G(U, V) = \{\varphi : U \to V \mid \varphi \text{ is a } G\text{-module homomorphism}\}.$$

   (a) Show that $\mathrm{Hom}_G(U, V)$ can be thought of as a vector space by defining an appropriate addition and scalar multiplication,

   (b) Give an explicit description of the vector space $\mathrm{Hom}_G(V, V)$ for the case when $V$ is irreducible.

## Recommended

Note that recommended problems come from our book. They have answers in the back (I will not grade them, though I am happy to talk about them).

1. Chapter 8: 4, 7
2. Chapter 9: 1, 2