Math 4140: Homework 4

Due: February 9, 2011

Required

1. Consider the two two-dimensional S_3-module given by $V = \mathbb{C}\text{-span}\{v_1, v_2\}$, where

$$v_1 * s_1 = -v_1, \quad v_2 * s_1 = v_2, \quad v_1 * s_2 = \frac{1}{2}v_1 + \frac{3}{2}v_2, \quad v_2 * s_2 = \frac{1}{2}v_1 - \frac{1}{2}v_2.$$

   and $U = \mathbb{C}\text{-span}\{u_1, u_2\}$ given by

$$u_1 * s_1 = u_1, \quad u_2 * s_1 = -u_2, \quad u_1 * s_2 = -\frac{1}{2}u_1 - \frac{3}{2}u_2, \quad u_2 * s_2 = -\frac{1}{2}u_1 + \frac{1}{2}u_2.$$

   (a) Find an explicit S_3-module isomorphism between the two modules (be sure to justify that it is a S_3-module homomorphism).

   (b) Find a matrix $a \in \mathrm{GL}_2(\mathbb{C})$ such that

$$(g\rho_U) = a^{-1}(g\rho_V)a, \qquad \text{for } g \in G.$$

2. Prove that if G is a finite group, then G has infinitely many different (nonequivalent) representations.

3. Prove that the characteristic of any field $\mathbb{F}$ is either a prime number or ∞.

4. (Harder) Decompose $\mathbb{C}S_3$ into irreducible modules. Be sure to give explicit bases for the irreducible modules in terms of the usual basis of $\mathbb{C}S_3$. Specify which of the irreducible modules are isomorphic, if any.

   Hint: No irreducible S_3-module has dimension more than 2.

Recommended

Note that recommended problems come from our book. They have answers in the back (I will not grade them, though I am happy to talk about them).

1. Chapter 7: 1, 3

2. Chapter 8: 1, 3