Math 4140: Homework 3

Due: February 2, 2011

Required

1. Show that
$$\langle s_1, s_2 \mid s_1^2 = s_2^2 = 1, (s_1 s_2)^n = 1 \rangle$$
is also a presentation for D_{2n}. That is, show how to write these new generators in terms of the other generators, and show that the old relations become these new ones.

2. (a) Show that every G-module that has dimension 1 is irreducible.
   (b) Find all the irreducible D_8-modules (as vector spaces over $\mathbb{C}$) that have dimension 1.

3. Consider the two-dimensional S_3-module $V = \mathbb{C}\text{-span}\{v_1, v_2\}$ given by
$$v_1 * s_1 = -v_1, \quad v_2 * s_1 = v_2, \quad v_1 * s_2 = \frac{1}{2}v_1 + \frac{3}{2}v_2, \quad v_2 * s_2 = \frac{1}{2}v_1 - \frac{1}{2}v_2.$$
   (a) Confirm that this action on V makes V a S_3-module by checking that it satisfies the relations of S_3.
   (b) Find the corresponding representation of S_3.
   (c) Determine whether or not V is irreducible. If so, prove it. If not, find a nonzero, proper submodule.
   (d) Consider the module $U = \mathbb{C}\text{-span}\{u_1, u_2\}$ given by
$$v_1 * s_1 = v_1, \quad v_2 * s_1 = -v_2, \quad v_1 * s_2 = -\frac{1}{2}v_1 - \frac{3}{2}v_2, \quad v_2 * s_2 = -\frac{1}{2}v_1 + \frac{1}{2}v_2.$$
   Determine whether U is isomorphic as an S_3-module to V.

Recommended

Note that recommended problems come from our book. They have answers in the back (I will not grade them, though I am happy to talk about them).

1. Chapter 4: 2, 3
2. Chapter 5: 1, 4