Math 4140: Homework 2

Due: January 26, 2011

1. For each $G \in \{S_3, C_8, D_8\}$, do the following:

   (a) Find a representation $\rho : G \to \mathrm{GL}_n(\mathbb{C})$ with $n \geq 2$ and $\rho(G) > 1$.

   (b) Construct the corresponding G-module V_ρ (that is, give a basis and describe the action of G on that basis).

   (c) Find a different basis for V_ρ and describe the corresponding new representation $\rho' : G \to \mathrm{GL}_n(\mathbb{C})$.

2. Let $\rho : G \to \mathrm{GL}_1(\mathbb{C})$ be a representation. Show that $G/\ker(\rho)$ is an abelian group.