Math 4140: Homework 11

Due: April 18, 2011

Required

1. Consider the class function θ of S_4 that has value

$$\theta(w) = \begin{cases} 1, & \text{if the cycle type of } w \text{ has evenly many rows,} \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Write θ as a linear combination of irreducible characters of S_4.

   (b) Is θ a character?

2. Consider the following table

1	1	1	1	1	1
3	-1	0	1	γ	$\bar{\gamma}$
3	-1	0	1	$\bar{\gamma}$	γ
5	2	0	0	-1	-1
7	-1	1	-1	0	0
8	0	-1	0	1	1

   where $\gamma = -\frac{1}{2} + i\frac{\sqrt{7}}{2}$. Show that it cannot be the character table of a group.

3. Consider the character table

G						
1	1	1	1	1	1	1
1	1	1	ω	ω^2	ω^2	ω
1	1	1	ω^2	ω	ω	ω^2
3	3	-1	0	0	0	0
2	-2	0	-1	-1	1	1
2	-2	0	$-\omega$	$-\omega^2$	ω^2	ω
2	-2	0	$-\omega^2$	$-\omega$	ω	ω^2

   where $\omega = e^{2\pi i/3}$. Find the following for G

   (a) $|G|$

   (b) The number of conjugacy classes of G

   (c) The sizes of the conjugacy classes of G.

Recommended

Chapter 15. 1, 2

Chapter 16. 1, 2, 4