Math 4140: Homework 10

Due: April 6, 2011

Required

1. We computed the irreducible characters of D_{2n} and you've computed the conjugacy classes of D_{2n} on a previous homework assignment. Compute the character tables of D_6 and D_8. Note that you can look up the tables in the book, but a full solution requires actual computations of the values using the irreducible representations.

2. Prove that for abelian groups, two modules are isomorphic if and only if their character values are equal.

3. Consider the permutation module of S_n, $V = \mathbb{C}\text{-span}\{v_1, \ldots, v_n\}$ given by $v_i * w = v_{iw}$ for $w \in S_n$. We have the submodule

$$U = \mathbb{C}\text{-span}\{v_1 - v_2, v_2 - v_3, \ldots, v_{n-1} - v_n\}.$$

   Use this basis to show that the character value $\chi_U(w)$ is the number of fixed points of w minus 1.

4. By computing the character values of each module, show that

$S_5^{\ydiagram{3,1,1}}$ and $S_5^{\ydiagram{2,2,1}}$

   are not isomorphic.

Recommended

Chapter 12. 2, 4

Chapter 13. 2, 5, 7