Math 4140: Homework 6

Due February 25, 2009

1. Let R be a root system with a base B.

   (a) Let $\gamma \in R$. Show that the set

   $$s_\gamma(B) = \{s_\gamma(\alpha) \mid \alpha \in B\}$$

   is also a base for R.

   (b) Deduce that if W is the Weyl group of R, then $w(B)$ is a base for R for any $w \in W$.

2. Let W be the Weyl group of a root system R. Show that for $w \in W$ and $\alpha \in R$,

   $$w s_\alpha w^{-1} = s_{w(\alpha)}.$$

3. For each of the following, show that it is an irreducible root system in the vector space spanned by the vectors, and find a base.

   (a) $R(B_n) = \{\pm e_k, \pm(e_i + e_j), \pm(e_i - e_j) \mid 1 \leq k \leq n, 1 \leq i < j \leq n\}$

   (b) $R(C_n) = \{\pm 2e_k, \pm(e_i + e_j), \pm(e_i - e_j) \mid 1 \leq k \leq n, 1 \leq i < j \leq n\}$

   (c) $R(D_n) = \{\pm(e_i + e_j), \pm(e_i - e_j) \mid 1 \leq i < j \leq n\}$

4. For one of the three root systems of Problem 3, do the following.

   (a) For $\alpha, \beta \in B$ (the base you found in Problem 3), find the smallest m such that

   $$(s_\alpha s_\beta)^m = 1.$$

   (b) Do you see any relationship between m and the angle between α and β? Can you formulate this in a formula for m?