

### Math 3170: Homework 3

1. Prove

$$3^n = \sum_{k=0}^n 2^k \binom{n}{k}$$

in two different ways.

2. Prove

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2$$

in two different ways.

3. What is the coefficient of  $x^n$  in the power series  $\sqrt[3]{1-2x}$ ?

4. (a) What is the power series for

$$\frac{1}{(1-x)^2}?$$

- (b) Compute the first ten terms of the sequence

$$h_n = h_{n-1} + h_{n-2} - h_{n-3},$$

with  $h_0 = 0$ ,  $h_1 = 1$ ,  $h_2 = 1$ .

- (c) Use generating functions to find a closed formula for  $h_n$  (although it is easy to do so without generating functions).

5. Let  $a, b, n \in \mathbb{Z}_{\geq 0}$  such that  $a + b \leq n$ .

- (a) Show that

$$(1-x)^{-a-1} = \sum_{j=0}^{\infty} \binom{a+j}{j} x^j.$$

- (b) Deduce that

$$\binom{n+1}{a+b+1} = \sum_{k=0}^n \binom{k}{a} \binom{n-k}{b}.$$

[Hint: Available.]