

Math 3170: Homework 7

Due: October 24, 2012

1. How many 2-digit positive integers are relatively prime to both 2 and 3?

2. For $m \in \mathbb{Z}_{\geq 1}$, let

$$\phi(m) = \#\{j \in \{1, 2, \dots, m\} \mid \gcd(m, j) = 1\}.$$

Let p, q, r be prime numbers. What is $\phi(pqr)$?

3. (a) Let

$$f_{k,n} = \# \left\{ \begin{array}{l} \text{set partitions of } \{1, 2, \dots, n\} \\ \text{into } k \text{ subsets that contain} \\ \text{at least 2 elements} \end{array} \right\}.$$

Find and prove a formula for $f_{k,n}$ in terms of the Stirling numbers of the second kind.

(b) Let

$$f_n = \# \left\{ \begin{array}{l} \text{set partitions of } \{1, 2, \dots, n\} \\ \text{into subsets that contain} \\ \text{at least 2 elements} \end{array} \right\}.$$

Find and prove a formula for f_n in terms of the Bell numbers.

4. Give a self-contained definition of a simple, directed graph.