Math 3170: Homework 3

Due: September 19, 2012

1. Prove
$$3^n = \sum_{k=0}^{n} 2^k \binom{n}{k}$$
in two different ways.

2. Prove
$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$
in two different ways.

3. What is the coefficient of x^n in the power series $\sqrt[3]{1-2x}$?

4. (a) What is the power series for
$$\frac{1}{(1-x)^2}?$$
(b) Compute the first ten terms of the sequence
$$h_n = h_{n-1} + h_{n-2} - h_{n-3},$$
with $h_0 = 0$, $h_1 = 1$, $h_2 = 1$.
(c) Use generating functions to find a closed formula for h_n (although it is easy to do so without generating functions).

5. Let $a, b, n \in \mathbb{Z}_{\geq 0}$ such that $a + b \leq n$.
(a) Show that
$$(1-x)^{-a-1} = \sum_{j=0}^{\infty} \binom{a+j}{j} x^j.$$
(b) Deduce that
$$\binom{n+1}{a+b+1} = \sum_{k=0}^{n} \binom{k}{a} \binom{n-k}{b}.$$
[Hint: Available.]