Math 3170: Homework 2

Due: September 8, 2010

1. Let T be a right triangle with side lengths $1/\sqrt{2}$ and hypotenuse length 1. Show that in any set of 17 points placed in T, there must be two that are at most $1/4$ apart.

   Hint: This is similar to one of the exercises in Chapter 1.

2. Find a closed formula for
$$\sum_{i=1}^{n} i(i+1),$$
   and prove the formula is correct.

3. Show that for each $n \in \mathbb{Z}_{\geq 1}$, one can partition a triangle into $3n + 1$ similar triangles.

4. How many 4-digit positive integers are there where all the integers are different?

5. There are n couples invited to a banquet. Suppose there are some number of speeches given during the banquet, so that at most one member of each couple gives a talk. How many different sets of speakers could the evening have?

6. Andy and Brenda are playing dice. Andy throws 4 dice simultaneously. Andy wins if a 6 appears, and Brenda wins if no 6 appears. Who is more likely to win?

7. How many ways are there of placing pawns in an $n \times n$ chessboard such that every row and every column of the board has an even number of pawns?