Math 3170: Homework 8

Due: October 27, 2010

1. (a) Find all nonisomorphic simple graphs on 4 vertices.
   (b) How many nonisomorphic arbitrary graphs are there on four vertices?

2. For which n can one partition the edges of K_n into subsets where each subset comes from a closed Hamiltonian path.

3. The n dimensional hypercube Q_n is the simple graph with vertices

$$V = \{(a_1, a_2, \ldots, a_n) \in \{0,1\}^n\},$$

   and an edge between $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ if

$$\#\{1 \leq j \leq n \mid a_i = b_i\} = n - 1.$$

   (a) How many vertices does Q_n have?
   (b) What are the degrees of the vertices?
   (c) Why is Q_n called a hypercube?
   (d) Show that for $n \geq 2$, Q_n has a closed Hamiltonian path.

4. The *girth* of a graph G is the number of edges in the smallest closed path of a graph.
   (a) Find all the simple graphs on 4 vertices with girth 3.
   (b) Let G be a simple graph with girth 5 such that each vertex v has degree at least d. Show that G has at least $d^2 + 1$ vertices.
   Hint: Fix a specific vertex, and look at all the vertices up to two steps away.