Math 3140: Homework 8

Due: Wednesday, October 29

A. 13.5 Let G be a group of order $4n+2$. Show that G contains a group of order $2n+1$. Hint: Use Cauchy's Theorem, Cayley's Theorem, and Exercise 6.6.

14.2 Find the conjugacy classes for D_n for all n (be careful to distinguish between different cases).

14.3 Suppose $\varphi : G \to H$ is an isomorphism of groups, and suppose C is a conjugacy class of G. Show that $\varphi(C)$ is a conjugacy class of H.

14.5 Prove that the 3-cycles of A_5 form a single conjugacy class. Find two 5-cycles which are *not* conjugate in A_5 (though they are conjugate in S_5).

14.10 Find the center of D_n for all n.

B. (a) Suppose R and S are rings. Give a careful definition of what you think a *ring homomorphism* should be.

(b) Let $\varphi : R \to S$ be a ring homomorphism. Define the *kernel* of φ to be the set

$$\ker(\varphi) = \{r \in R \mid \varphi(r) = 0\}.$$

Show that for all $r \in R$ and $k \in \ker(\varphi)$,

$$rk, kr \in \ker(\varphi).$$

(c) An *ideal* of a ring R is a subring I such that

$$ri, ir \in I \qquad \text{for all } i \in I,\ r \in R.$$

Find a multiplicative function and an additive function that makes

$$R + I = \{r + I \mid r \in R\}$$

a ring.

(d) Show that $I \subseteq R$ is an ideal if and only if there exists a ring homomorphism $\varphi : R \to S$ for some ring S such that $I = \ker(\varphi)$.