Math 3140: Homework 7

Due: Wednesday, October 22

A. Let $C_r = \langle x \rangle$ be the cyclic group with r elements (but written with multiplication, rather than addition). Let

$$W_{r,n} = \left\{ a \in M_n(C_r \cup \{0\}) \;\middle|\; \begin{array}{l} a \text{ has exactly one nonzero entry} \\ \text{in every row and every column} \end{array} \right\}.$$

(a) Show that $W_{r,n}$ is a group.

(b) Show that $W_{2,2} \cong D_4$.

(c) What more familiar groups are $W_{1,n}$ and $W_{r,1}$ isomorphic to?

(d) What is the order of $W_{r,n}$?

B. 11.4 Suppose $|G|$ is the product of two distinct primes. Show that any proper subgroup of G must be cyclic.

11.7 Suppose $n \in \mathbb{Z}_{\geq 1}$ and m divides $2n$. Show that D_n contains a group of order m.

11.8 Does A_5 contain a subgroup of order m for every m that divides $|A_5| = 60$?

12.4-5 Find examples of a group G and a subgroup H such that the following sets are **not** equivalence relations:

(a) $\{(x, y) \mid xy \in H\}$,

(b) $\{(x, y) \mid xyx^{-1}y^{-1} \in H\}$.

12.8 Let H be a subgroup of a group G.

(a) Show that if $|G| = 2|H|$, then $gH = Hg$ for all $g \in G$.

(b) Show that $gH = Hg$ for all $g \in G$ if and only if $ghg^{-1} \in H$ for all $h \in H$, $g \in G$.

13.2. Suppose G is abelian with $|G|$ a product of distinct primes. Show that G is cyclic.

13.4. Classify the groups of order 10.