Math 3140: Homework 8

Due: Wednesday, October 30

A. 13.5 Let G be a group of order $4n+2$. Show that G contains a group of order $2n+1$. Hint: Use Cauchy's Theorem, Cayley's Theorem, and Exercise 6.6.

14.2 Find the conjugacy classes for D_n for all n (be careful to distinguish between different cases).

14.3 Suppose $\varphi : G \to H$ is an isomorphism of groups, and suppose C is a conjugacy class of G. Show that $\varphi(C)$ is a conjugacy class of H.

14.5 Prove that the 3-cycles of A_5 form a single conjugacy class. Find two 5-cycles which are *not* conjugate in A_5 (though they are conjugate in S_5).

14.10 Find the center of D_n for all n.

B. (a) Show that if $w \in S_n$, then both w and w^{-1} are in the same conjugacy class. Find an example of a group for which this is not true.

(b) Suppose G is a matrix group. Show that if $g, h \in G$ are in the same conjugacy class, then $\det(g) = \det(h)$.

C. (a) Given a ring R, give a definition of a *subring generated by a subset* $S \subseteq R$.

(b) Recall, that $M_{n\times n}(\mathbb{Z})$ is a ring. Find the subring generated by the element

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$