

# Math 3140: Homework 12

Due: Wednesday, December 11

- A. 18.1 Use orbit counting methods to find the number of distinct ways to paint the edges of a cube with two colors.
- 18.7. How many different ways are there of coloring the vertices and edges of a regular hexagon using red, black or yellow for the edges and black or white for the vertices?
- B. 20.1 (a) Show that if  $|G| = 126$ , then  $G$  has a nontrivial proper normal subgroup.  
(b) Show that if  $|G| = 1000$ , then  $G$  is not simple.  
(c) Suppose  $|G| = p^k m$  where  $p$  is prime and  $p$  does not divide  $m$ . Prove that if  $p > m$ , then  $G$  is not simple.
- (1) Prove that if  $G$  is abelian and simple, then  $G \cong \mathbb{Z}_p$  for some prime number  $p$ .
- 20.3. (a) Prove that if all the Sylow subgroups are normal, then  $G$  is isomorphic to the direct product of of its Sylow subgroups.  
(b) If you know that  $G$  is abelian, and  $|G| = 154000$ , then what do you know about  $G$ ?
- 20.7. Classify the groups of order  $p^2 q$  if  $p$  is not congruent to  $\pm 1$  modulo  $q$  (and  $p \neq q$  are prime).