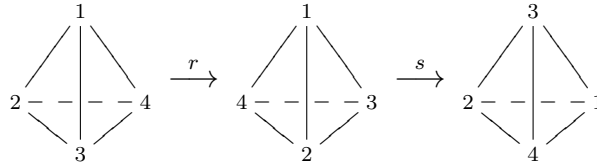


Math 3140: Homework 1

Due: Wednesday, September 4

A. Figure 1.4 essentially describes two symmetries of the tetrahedron:



- 1.3 Adopt the notation of Figure 1.4. Show that the axis of the composite rotation srs passes through the vertex 4, and that the axis of $rsrr$ is determined by the midpoints of edges 12 and 34.
 - 1.4 Having completed 1.3, Express each of the twelve rotational symmetries of the tetrahedron in terms of r and s .
 - 1.5 Again with the notation of Figure 1.4, check that $r^{-1} = rr$, $s^{-1} = s$, $(rs)^{-1} = srr$, and $(sr)^{-1} = rrs$.
 - 1.9 Find all plane symmetries (rotations and reflections) of a regular pentagon.
- B.
- 2.5 A function from the plane to itself which preserves the distance between any two points is called an *isometry*. Prove that an isometry must be a bijection and check that the collection of all isometries of the plane forms a group under composition of functions.
 - 2.6 Show that the collection of all rotations of the plane about a fixed point P forms a group under composition of functions. Is the same true of the set of all reflections in lines which pass through P ? What happens if we take all the rotations and all the reflections?
 - 2.7 Let x and y be elements of a group G . Prove that G contains elements w, z which satisfy $wx = y$ and $xz = y$, and show that these elements are unique.
 - 2.8 If x and y are elements of a group, prove that $(xy)^{-1} = y^{-1}x^{-1}$.
- C. What are all the groups with 4 elements? 5 elements?