Math 3140: Homework 8

Due: Wednesday, October 27

A. 14.2 Find the conjugacy classes for D_n for all n (be careful to distinguish between different cases).

14.3 Suppose $\varphi : G \to H$ is an isomorphism of groups, and suppose C is a conjugacy class of G. Show that $\varphi(C)$ is a conjugacy class of H.

14.5 Prove that the 3-cycles of A_5 form a single conjugacy class. Find two 5-cycles which are *not* conjugate in A_5 (though they are conjugate in S_5).

(4) Show that if $w \in S_n$, then both w and w^{-1} are in the same conjugacy class. Find an example of a group for which this is not true.

14.10 Find the center of D_n for all n.

(6) Suppose G is a matrix group. Show that if $g, h \in G$ are in the same conjugacy class, then $\det(g) = \det(h)$.

B. 13.2 Suppose G is abelian and suppose $|G| = p_1 p_2 \cdots p_r$ factors into distinct primes p_j. Show that G is abelian.

13.5 Let G be a group of order $4n+2$. Show that G contains a group of order $2n+1$.

Hint: Use Cauchy's Theorem, Cayley's Theorem, and Exercise 6.6.

C. Find an example groups G, H and a homomorphism $\varphi : G \to H$ such that φ is neither surjective nor injective.