Math 3140: Homework 5

Due: Wednesday, September 28

A. 7.2 Verify that the set of integers $\{1, 3, 7, 9, 11, 13, 17, 19\}$ form a group under multiplication modulo 20. Explain why this group is not isomorphic to $\mathbb{Z}_8$.

7.9. Suppose G is cyclic with generator $x \in G$. Show that if $\varphi : G \to H$ is an isomorphism, then φ is completely determined by $\varphi(x)$. Show that $H = \langle \varphi(x) \rangle$.

B. 8.7 Using Cayley's Theorem, explicitly find an isomorphic copy of D_3 inside S_6.

8.10 For each $w \in S_n$, let $\tilde{w} \in S_{2n}$ be the permutation given by

$$\tilde{w}(j) = \begin{cases} w(j), & \text{if } 1 \leq j \leq n, \\ w(j-n)+n, & \text{if } n+1 \leq j \leq 2n. \end{cases}$$

(a) Describe the relationship between the diagram of w and the diagram of $\tilde{w}$.

(b) Show that the function that sends $w \mapsto \tilde{w}$ is an isomorphism between S_n and a subgroup of A_{2n}.

8.11 Let G be the full symmetry group of a regular tetrahedron T, and adopt the notation of Figure 7.2 in the book. Find the symmetry q of T which induces the transposition $(1, 2)$ of the vertices, and show that qr induces the 4-cycle $(1, 2, 3, 4)$. Check that qr is neither a rotation nor a reflection, but is the product of three reflections. Count the symmetries of T and prove that G is isomorphic to S_4.

C. 10.1 Show that if $G \times H$ is cyclic, then both G and H are cyclic.

10.2 Show that $\mathbb{Z} \times \mathbb{Z}$ and $\mathbb{Z}$ are not isomorphic.