Project 2

Due: November 9, 2007

Recommended length: 3-5 pages.
Format: typed.

The goal of this assignment is to examine the behavior of in and out shuffles. Suppose that you have a deck of $2n$ cards. Let $i \in S_{2n}$ denote the permutation that gives an in-shuffle, and let $o \in S_{2n}$ denote the permutation that gives the out-shuffle. Let

$$B_n = \left\{ \begin{array}{c} n \times n \text{ matrices with entries in } \{-1,0,1\} \\ \text{and exactly one nonzero entry in} \\ \text{every row and in every column} \end{array} \right\}.$$

For $b \in B_n$, let $\bar{b} \in W_n$ be the same matrix as b but with -1's replaced by 1's. The goals of the project are to

(1) Prove the following two results.

Theorem. *The group $\langle i, o\rangle$ is isomorphic to a subgroup of B_n.*

Corollary. *The order of $\langle i, o\rangle$ is less than or equal to $2^n n!$.*

(2) Investigate the behavior of the three following functions.

$$\begin{array}{rcl} \sigma : B_n & \longrightarrow & \{-1,1\} \\ b & \mapsto & \det(b) \end{array} \qquad \begin{array}{rcl} \bar{\sigma} : B_n & \longrightarrow & \{-1,1\} \\ b & \mapsto & \det(\bar{b}), \end{array} \qquad \begin{array}{rcl} \sigma \times \bar{\sigma} : B_n & \longrightarrow & \{-1,1\} \\ b & \mapsto & \det(b)\det(\bar{b}). \end{array}$$

In particular, show that each is a homomorphism, and describe their kernels.

Note that this is a writing assignment, so the main focus should be on clearly communicating the ideas in the proof. I recommend looking at your favorite mathematics texts and trying to emulate their style. I also suggest you have another member of the class read through a draft before handing it in.