Introduction to LaTeX: Fundamentals of counting

Math 2001

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1 Introduction

This is an introduction to both LaTeXand the fundamentals of counting. So we can see how to write nicely formatted mathematics, and have lecture notes for this lecture.

To start a new paragraph, just hit enter twice. No matter how many spaces I use, the program will normalize the spacing, and justify the margins.

For counting, we want to break down the process of counting into predictable steps, because counting is *hard*.

2 Preliminaries

There are different ways to write math. In line mathematics gets surrounded by dollar signs. For example, $x = 3^3$. Alternatively, we could display some mathematics with double-dollar signs, as in

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

For a rational function,

$$\frac{x^2 - 2x - 3}{\sqrt{4\pi - 7}} \quad \text{vs} \quad \frac{x^2 - 2x - 3}{\sqrt{4pi - 7}}.$$

Try κ. For example, Θ vs. θ vs. ϑ.

The goal in counting is to carefully keep track of the choices made in constructing a generic element of the set. I use two different crutches to help keep track:

(OYC) "Or You Could" corresponds to addition.

(ATY) "And Then You" corresponds to multiplication.

For example,

Question 2.1. *How many 5 card flushes are there in a standard 52 card deck?*

2.1 Approach 1

We construct a generic 5 card flush by

Option 1. We could pick 5 $\heartsuit$ ($\binom{13}{5}$ choices).

Option 2. Or we could pick 5 ♠ ($\binom{13}{5}$ choices).

Option 3. Or we could pick 5 ♢ ($\binom{13}{5}$ choices).

Option 4. Or we could pick 5 ♣ ($\binom{13}{5}$ choices).

Thus, in total, to construct a generic element we have

$$\binom{13}{5} + \binom{13}{5} + \binom{13}{5} + \binom{13}{5}$$

many choices, so there are $4\binom{13}{5}$ 5-card flushes in a deck of 52 cards.

2.2 Approach 2

We construct a generic 5-card flush by

Step 1. Pick a suit ($\binom{4}{1}$ choices).

Step 2. And then you choose a 5-card flush in that suit ($\binom{13}{5}$ options).

Thus, in total, we have

$$\binom{4}{1} \cdot \binom{13}{5}$$

choices and there are $\binom{4}{1}\binom{13}{5}$ different 5-card flushes.

Question 2.2. *How many sets of two 5-card flushes are there in a standard 52 card deck?*

Option 1. We choose two flushes from one suit. ($\binom{4}{1}$ choices).

Step 1.1. We choose 10 cards ($\binom{13}{10}$ choices).

Step 1.2. And then we separate into two sets of 5 ($\binom{9}{4}$ choices).

Option 2. We choose flushes from two different suits. ($\binom{4}{2}$ choices).

Step 2.1. Pick one of the two ($\binom{2}{1}$ choices).

Step 2.2. Pick a flush in that suit ($\binom{13}{5}$ choices).

In total we get

$$\binom{4}{1}\binom{13}{10}\binom{9}{4} + \binom{4}{2}\binom{2}{1}\binom{13}{5}.$$

3 Main results

This section examines some of the math behind (OYC) and (ATY).

A *set partition* of a set B is a set of nonempty subsets $\{A_1, \ldots, A_\ell\} \subseteq P(B)$, such that

1. $B = A_1 \cup A_2 \cup \cdots \cup A_\ell$,
2. for $1 \leq i < j \leq \ell$, we have $A_i \cap A_j = \emptyset$.

Theorem 3.1 (OYC theorem)**.** *Let* $\{A_1, A_2, \ldots, A_\ell\}$ *be a partition of* B*. Then*

$$|B| = |A_1| + |A_2| + \cdots + |A_\ell|.$$

Proof. Unfortunately, we will need induction to prove this theorem. □

$\forall, \exists \in$

4 Appendix