

Math 2001: PHW 1

1. If we assume that

“There is always sun when the wind is in the East,”

then which of the following statements are true:

- (a) If it’s sunny, then the wind must be in the East,
- (b) If the wind isn’t in the East, then it must be overcast,
- (c) If it’s overcast, then the wind must not be in the East.

Give an explanation of your answers.

2. Consider the two statements

- “For each BLAH there exists BLOOF.”
- “For all BLAH there exists BLOOF.”

Either explain why they are the same, or give example BLAHs and BLOOFs to show how they are different. Warning: Do not be tempted to treat these sentences as being written with logical quantifiers (if you know what that means).

3. Consider the statement

Theorem. *Suppose G is a connected graph. If G has an Eulerian circuit, then every vertex of G has even degree.*

Without necessarily understanding what this says, answer the following.

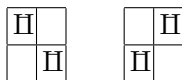
- (a) What are the assumptions of the statement (if any)?
- (b) What are the conclusions of the statement (if any)?

4. Analyze tic-tac-toe.

- (a) Is there always a winner?
- (b) What is an optimal strategy?
- (c) Does the optimal strategy depend on whether you go first or second?

Justify your answers as well as possible.

5. How many ways are there of placing 4 non-attacking rooks on an 4×4 chess-board? What about 5 non-attacking rooks on a 5×5 chessboard. Example: there are 2 ways of placing 2 non-attacking rooks on a 2×2 chessboard,



Can you formulate a guess for how many ways there are to place n non-attacking rooks on an $n \times n$ chessboard?

6. There’s a party with 7 guests. If each guest shakes the hand of every other guest, how many handshakes are there?