# Introduction to LaTeX:
# A journey via proof by contradiction

Math 2001

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## 1 Introduction

This document is simultaneously an introduction to LaTeX, and a lecture on "proof by contradiction."

There are two modes in tex. Math mode and text. I'm writing in text mode and you will see that even if I add extra spacing, tex will fix it. In math mode, you surround the math in dollar signs, and it turns $allletters into math symbols$ and it removes spaces. Now the element $a$ looks different than the letter a. You can also display math easily

$$using double dollar signs.$$

## 2 Preliminaries

Proof by contradiction is a technique for proving statements that works as follows.

**Assumptions.** We assume all the assumptions are true, and we assume the conclusions are false.

**To show.** Something turns out to be both true and false.

## 3 Main examples

We will now prove a couple of theorems using proof by contradiction. First consider the game of 2-move chess. This is the same is played by each player getting two turns at a time.

**Theorem 3.1.** *In the game of 2-move chess, the first player has a non-losing strategy.*

*Proof.* Assume the second player has a winning strategy. The first player moves the knight out, and then moves the knight back. Now player one pursues the player 2 winning strategy. This contradicts player 2 having had a winning strategy. □

**Remarks.**

1. We do not know this non-losing strategy.

2. What other games does this argument apply to?

**Theorem 3.2.** $\sqrt{2} \notin \mathbb{Q}$.

*Proof.* Suppose

$$\sqrt{2} = \frac{a}{b}$$

for some $a, b \in \mathbb{Z}$. We may assume that $a/b$ is reduced. Then

$$\sqrt{2}b = a \qquad \text{so} \qquad 2b^2 = a^2.$$

Since $a^2$ is even, we know that $a$ is even (recall, we proved that "$a$ and $b$ odd implies $ab$ odd"). Thus, $a = 2c$ for some $c \in \mathbb{Z}$, and

$$2b^2 = 4c^2 \qquad \text{so} \qquad b^2 = 2c^2.$$

By the same argument, $b$ must be even. This contradicts $a/b$ reduced. □

**Theorem 3.3.** *It is impossible to list all infinite binary sequences.*

*Proof.* Suppose you could. Do it. List them out. So we have something like

$$\begin{array}{cccccc} \underline{0} & 1 & 0 & 0 & 1 & \cdots \\ 1 & \underline{1} & 1 & 1 & 1 & \cdots \\ 0 & 1 & \underline{1} & 0 & 1 & \cdots \\ 1 & 1 & 0 & \underline{0} & 1 & \cdots \\ \vdots & & & & \ddots & \end{array}$$

Now underline the diagonal terms. Create a new sequence by reading the underlined terms but changing all the values, so

$$1\,0\,0\,1\,\cdots.$$

This sequence is not in my list. The first entry is different from the first sequence on my list, the second entry is different from the second sequence on my list, ..., the $k$th entry is different from the $k$th sequence on my list, and so on. This contradicts that we listed all the binary sequences. □

# 4 Appendix

Other things to do. $\cap$, $\cup$, $\wedge$ and $\vee$, $\forall$ and $\exists$, $\sim$,

$$\overline{(A \cap B) \times C}.$$

$\implies$, $\{\}$ and

$$\left(\sum_{k=1}^{\infty} k^2\right)$$