

Math 2001: Homework 8

Due: October 29, 2008

Give complete justifications for all your answers.

Problem 1

1. Describe the patterns in Pascal's triangle if you replace each number by its congruence class in \mathbb{Z}_2 .
2. Describe the patterns in Pascal's triangle if you replace each number by its congruence class in \mathbb{Z}_3 .

Problem 2

In \mathbb{Z}_{11} decide if the following expressions make sense. Evaluate them if they do, and explain why they do not if they do not.

1. $[10]^{36} - [1]$.
2. $\frac{[5][7] - [3]}{[2]}$.
3. $\sqrt{[3]}$.
4. $\sqrt{[7]}$.

Hint: All but one make sense.

Problem 3

Recall that the equation

$$x^2 + 1 = 0$$

has no solutions that are real numbers (and so $x^2 + 1$ cannot be factored unless we allow complex numbers).

1. Show that there exists $x \in \mathbb{Z}$ such that

$$x^2 + 1 \equiv 0 \pmod{5}.$$

Thus, over \mathbb{Z}_5 , find $a, b \in \mathbb{Z}$ such that $[x]^2 + [1] = ([x] - [a])([x] - [b])$.

2. Find a positive integer n such that

$$x^2 + 1 \equiv 0 \pmod{n}$$

has no solution.

3. Given $n \in \mathbb{Z}_{>1}$, give a criterion for determining whether or not $x^2 + 1 \equiv 0 \pmod{n}$ has a solution.