Math 2001: Homework 4

Due: September 24, 2008

Give complete justifications for all your answers.

Problem 1

From the book:

1. Construct the set of positive integers which give a remainder of 3 when divided by 4 using set-builder notation.

2. Let
$$X = \{n \in \mathbb{Z} \mid 1 \leq n \leq 16\}, \quad A = \{5, 9, 13\}, \quad B = \{3, 7, 11, 15\}.$$
Find $A \times B$, $A \cup B$, $A \cap B$, $A - B$, A^c and B^c.

Problem 2

Give examples of the following, or explain why they don't exist.

1. An infinite set with a finite number of subsets,

2. A finite set with an infinite number of subsets,

3. A finite set with the same number of subsets and elements.

Problem 3

1. Let A be a set, and let B be the set of subsets of A. Is $A \in B$ or $A \subseteq B$? Justify your answer.

2. What is the number of subsets of the set $\{\{1, 2, 3\}, \{1\}, \{1, 4\}, \{1, 4, 5, \{1, 2\}\}, \{1, 2, 3, 4\}\}$?

3. What is the number of subsets of $\{a, b, c, d, e, f\}$ which all contain c? Generalize by determining how many subsets of $\{1, 2, \ldots, n\}$ contain 1. Prove by induction.

4. Prove directly that for $0 < k < n$,
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k},$$
using only the fact that
$$\binom{n}{k} = \text{the number of subsets of } \{1, 2, \ldots, n\} \text{ with } k \text{ elements.}$$

5. (Harder) What is the size of the set
$$\{(m_1, m_2, \ldots, m_k) \mid m_1, m_2, \ldots, m_k \in \{1, 2, 3, \ldots, n\}, m_1 + m_2 + \cdots + m_k = n\}?$$
Your answer should depend on n and k. For example, if $n = 3$, then the set is

	The set
$k = 1$	$\{(3)\}$
$k = 2$	$\{(2, 1), (1, 2)\}$
$k = 3$	$\{(1, 1, 1)\}$

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