Math 2001: Homework P2

Due: September 10, 2008

Problem 1

Color all the odd numbers in Pascal's triangle red and all the even numbers blue. What pattern do you get? Describe it as precisely as you can.

Problem 2

Let $k, l, m, n \in \mathbb{Z}_{\geq 0}$ be such that $n = k + l + m$. The *trinomial coefficient* $\binom{n}{k, l, m}$ is given by the rules

(1) for $k + l = n$, $\binom{n}{k, l, 0} = \binom{n}{k, 0, l} = \binom{n}{0, k, l} = \binom{n}{k}$,

(2) $\binom{n}{k, l, m} = \binom{n-1}{k-1, l, m} + \binom{n-1}{k, l-1, m} + \binom{n-1}{k, l, m-1}$.

The following four questions use this definition.

(a) Describe the "triangle" of trinomial coefficients.

(b) Find, state, and prove a trinomial analogue to

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

(c) Find, state, and prove a trinomial analogue to

$$\binom{n}{k} = \begin{array}{l}\#\text{ of walks on a rooted binary tree starting}\\ \text{at the root with } n \text{ total steps using } k \text{ right steps.}\end{array}$$

(d) Define a multinomial coefficient $\binom{n}{k_1, k_2, \ldots, k_\ell}$, and state the analogues to (b) and (c) for multinomial coefficients (without proof).