

Review for Exam 2

1. In each part of the following problem, ESTIMATE to determine whether the first quantity is less than or greater than the second. EXPLAIN briefly your steps.

- (a) $161 \cdot 201$; 32,000 **Greater: $161 \cdot 201 > 160 \cdot 200 = 32,000$**
- (b) $29,331 \div 28$; 1,000 **Greater: $29331 \div 28 > 28000 \div 28 = 1,000$**
- (c) $29,331 \div 31$; 1,000 **Less: $29331 \div 31 < 31000 \div 31 = 1,000$**
- (d) $334 \cdot 301$; 100,000 **Greater: $334 \cdot 301 > 334 \cdot 300 = 100,200$**
- (e) $8387 \div 1777$; 5 **Less: $8387 \div 1777 < 8500 \div 1700 = 5$**
- (f) $998 \cdot 1002$; 1000^2 **Less: $998 \cdot 1002 = (1000 - 2)(1000 + 2) = 1000^2 - 2^2 < 1000^2$**

2. Use front end estimation to approximate each of the following sums to the nearest dollar. Explain your work.

\$3.41	\$2.15	\$11.04	\$3.41	\$2.15
\$2.63	\$1.17	\$0.31	\$8.23	\$0.20
\$0.49	\$3.67	\$2.36	\$3.21	\$10.74
\$1.13	\$0.17	\$0.11	\$3.21	\$0.33
\$0.63	\$2.88	\$0.31	\$1.77	\$12.20
\$1.11	\$7.97	\$2.22	\$1.23	\$10.00
+ \$0.12	+ \$4.77	+ \$33.11	+ \$8.88	+ \$42.54
\approx \$10.	\approx \$23.	\approx \$49.	\approx \$30.	\approx \$78.

3. Here are snowfall amounts in inches, for each of the 12 weeks of winter, in Frostbite Falls, Minnesota: 9, 9, 32, 31, 28, 11, 10, 11, 30, 8, 12, 11. Use clustering to estimate the total winter snowfall in Frostbite Falls. **About $8 \cdot 10 + 4 \cdot 30 = 200$ inches.**



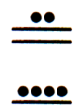

4. Evaluate each of the following Roman numerals:

- (a) MCMLXVII **1967**
- (b) MMMCDXLIX **3449**
- (c) CMXCIX **999**
- (d) MMMCCCXXXIII **3333**

5. Express each of the following as a Roman numeral:

- (a) 3097 **MMMXCVII**
- (b) 949 **CMXLIX**
- (c) 494 **CDXCIV**
- (d) 2222 **MMCCXXII**

6. Evaluate each of the following Mayan numerals:

- | | | | |
|---|---|---|---|
|  |  |  |  |
| (a) 2173 | (b) 119 | (c) 259 | (d) 7300 |

7. Express each of the following as a Mayan numeral:

(a) 20



(d) 252



(b) 361



(e) 129



(c) 3666 (hint: $3666 = 10 \cdot 18 \cdot 20 + 3 \cdot 20 + 6$)



(f) 7313 (hint: $7313 = 1 \cdot 20 \cdot 18 \cdot 20 + 5 \cdot 20 + 13$)



8. Evaluate each of the following Babylonian numerals:

(a) $\llcorner\llcorner\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown\blacktriangledown$ 27

(b) $\blacktriangleleft\blacktriangledown\blacktriangleleft\blacktriangledown$ 671

(c) $\blacktriangledown\blacktriangledown\blacktriangleleft\blacktriangledown\blacktriangledown$ 7802

9. Express each of the following as a Babylonian numeral:

(a) 20



(d) 252



(b) 361



(e) 129



(c) 3606 (hint: $3666 = 60^2 + 60 + 6$)



(f) 7313 (hint: $7313 = 2 \cdot 60^2 + 60 + 53$)



10. Convert each of the following numbers to base ten.

(a) 20_{seven}
14

(b) 100110_{two}
38

(c) 101_{nine}
82

(d) EE_{twelve}
143

11. Convert each of the following base ten numbers to the indicated base.

(a) 40_{ten} : base two
 101000_{two}

(b) 1110_{ten} : base five
 13420_{five}

(c) 567_{ten} : base four
 20313_{four}

(d) 242_{ten} : base twelve
 182_{twelve}

12. Write down, in the same base as is given, the number that's one larger than each of the following:

(a) 99999_{ten}
 100000_{ten}

(b) 111111_{two}
 1000000_{two}

(c) 44444_{five}
 100000_{five}

(d) $EEEEEE_{\text{twelve}}$
 100000_{twelve}

13. The odometer on your Math 1110 instructor's car records mileage in base six. (Well OK not *REALLY*, but let's pretend.) What did the odometer read just before it read 3,000,000? At that point (just before 3,000,000), how many miles (in base ten) had this car traveled? Hint: $3 \cdot 6^6 = 139968$. **It read 2,555,555, which means $3 \cdot 6^6 - 1 = 139967$ miles**

14. Suppose we want to estimate $x \div y$. If we round x down and y up, will our estimate be lower than the actual value, or higher, or might it be either? **It will be lower: rounding x down gives a smaller quotient, and rounding y up does the same thing.**

15. Perform each of the following additions or subtractions.

$$\begin{array}{r}
 1101_{\text{two}} \\
 + 101_{\text{two}} \\
 \hline
 10010_{\text{two}}
 \end{array}
 \quad
 \begin{array}{r}
 737_{\text{nine}} \\
 - 448_{\text{nine}} \\
 \hline
 278_{\text{nine}}
 \end{array}
 \quad
 \begin{array}{r}
 222_{\text{four}} \\
 + 333_{\text{four}} \\
 \hline
 1221_{\text{four}}
 \end{array}
 \quad
 \begin{array}{r}
 123_{\text{four}} \\
 - 33_{\text{four}} \\
 \hline
 30_{\text{four}}
 \end{array}
 \quad
 \begin{array}{r}
 90 \text{ TE}_{\text{twelve}} \\
 + \text{TE } 90_{\text{twelve}} \\
 \hline
 1807\text{E}_{\text{twelve}}
 \end{array}$$

16. Compute each of the following.

(a) $(-2)^8 \div (-2)^3 = -32$ (b) $-2^4 = -16$ (c) $(-2)^4 = 16$ (d) $(-1)^{10101} = -1$

17. Evaluate each of the following, or explain why it can't be evaluated.

(a) $(-10 \div 5)(-4) \div (-2) = -4$ (b) $(-10 \div 5)(-4) \div (-2 - (-2))$
can't be done (division by zero)
(c) $|-5| \cdot |-12| - |-2| = 58$ (d) $(-10 \div -5)(-4) \div (2 - (-2)) = -2$

18. Evaluate each sum or product by first grouping together compatible numbers.

(a) $39 + 41 + 22 + 12 + 61 + 59 + 78 + 4 = (39+61)+(41+59)+(22+78)+12+4=316$ (b) $2 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) \cdot 3 \cdot 7 = 210,000$

19. Evaluate each sum or product by trading off.

(a) $1175 + 2030 = 1200 + 2005 = 3205$ (b) $94 + 2706 = 100 + 2700 = 2800$
(c) $16 \cdot 18 = 8 \cdot 36 = 4 \cdot 72 = 2 \cdot 144 = 288$ (c) $12 \cdot 85 = 6 \cdot 170 = 3 \cdot 340 = 1020$

20. Answer each of the following without actually performing the division. Explain your answers in all cases.

(a) Is 23231 divisible by 23? no; 23|2323 so 23|23230, but 23/1 (b) Is $17!+3$ divisible by 4? no; 4|17! but 4/3
(c) Is 380,019 divisible by 19? yes; 19|38 so 19|380000; also, 19|19, so 19|380019 (d) Suppose n is even. Is $3n + 5$ divisible by 6? no; 6|3n but 6/5

21. Use divisibility tests to determine whether each of the following numbers is divisible by 3, 4, 6, 9, 11.

(a) 20,394 by 3,6,9,11 only (b) 362,880 by 3,4,6,9 only (c) 1,393,194 by 3,6,11 only
(d) 111,111,111 by 3,9 only (e) 1,111,111,111 by 11 only

22. (a) If $a \nmid b$ and $a \nmid c$, is it necessarily true that $a \nmid (b + c)$? Explain. No: e.g. 5/7 and 5/8, but 5|(7 + 8)

(b) If $a|c$ and $b|c$, is it always true that $ab|c$? Explain. No: e.g. 3|12 and 6|12, but 3 \cdot 6/12

23. Fill in the blank in the number 987,6____4 so that the result is divisible by:

(a) 4 987624 (b) 6 987624 (c) 9 987624 (d) 11 987624

24. Using divisibility tests only, explain why 9,790 is divisible by: (a) 2 because 2|0; (b) 5 because 5|0; (c) 11 because 11|(9 - 7 + 9 - 0) = 11; (d) 110 because 2|9790, 5|9790, 11|9790, 110 = 2 \cdot 5 \cdot 11, and 2, 5, and 11 are relatively prime.