Math 1110: The Spirit and Uses of Mathematics **Fall 2008**

Review for Exam 1

1. Find $45 + 46 + \cdots + 200 + 201$. Show your work.

2. If $n(A) = 7$, $n(B) = 8$, and $n(C) = 9$, what is the greatest and least number of elements in each of the following?

(a) $A \cap B \cap C$ **(b)** $A \cup B \cup C$

(c) $(A \cup B) - C$ **(d)** $A \cup (B - C)$

3. Each part of this problem contains both an expression and some equations. First, evaluate the expression. Then, put parentheses, if necessary, into the equation to make it true.

(a) $3 \times 2 + 4 - 6 = 4$ $3 \times 2 + 4 - 6 = 12$ $3 \times 2 + 4 - 6 = 0$

(b) $4 \times 3 + 6 \div 2 = 15$ $4 \times 3 + 6 \div 2 = 18$ $4 \times 3 + 6 \div 2 = 24$ $4 \times 3 + 6 \div 2 = 9$

4. Use a block model (that is, an area model) to illustrate the fact that $(a + b)(a + c) = a^2 + ba + ac + bc$.

5. List all elements of each of the following sets.

(a) $\{x | x$ is an even number between 1 and 11$\}$ **(b)** $\{x | x$ is a circle and x is a square$\}$

6. List all subsets of each of the following sets.

(a) {Your 1110 instructor}

(b) $\{\alpha, \beta, \gamma\}$

(c) $\{x | x$ is a circle and x is a square$\}$

7. Describe all one-to-one correspondences between each given pair of sets.

(a) $\{1, 2\}$ and $\{u, v\}$ **(b)** $\{1, 2, 3\}$ and $\{u, v\}$ **(b)** $\{1, 2, 3\}$ and $\{u, v, w\}$

8. For each of the following, determine whether the sequence is arithmetic, geometric, or neither. Write down the next four terms in the sequence. Also find the 50th and the nth term in the sequence.

(a) $2, 4, 6, 8, 10$ **(b)** $0, 2, 4, 6, 8, 10$ **(c)** $1, 6, 11, 16, 21$

(d) $1, 5, 25, 125, 625$ **(e)** $1, 1, 1, 1, 1, 1$ **(f)** $1, 2, 4, 8, 16$

(g) $0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1$ **(h)** $-5, 5, -5, 5, -5$

9. For each of the following, draw the next three figures in the sequence. Also draw the 100th figure in the sequence.

(a)

(b)

(c)

10. The Yo, Fuggetaboutit Pizza and Mathematics Club in New York City has a reunion and all 60 members attend. Available Pizza toppings at the reunion are mushroom, onion,

and Tootsie Roll. All members have some pizza; 9 have pizza with no toppings; 35 have (at least) mushroom; 23 have (at least) onion; 20 have (at least) mushroom and Tootsie Roll; 7 have (at least) onion and Tootsie Roll; 9 have (at least) mushroom and onion; 2 have all three toppings.

(a) Draw a properly labeled Venn diagram depicting the situation. (Describe the major sets in question, including the universe, using set builder notation.)

(b) How many members had Tootsie Roll only on their pizza?

(c) In terms of the sets from part (a), and unions and intersections and so on, describe the set discussed in part (b) of this problem.

11. Write down a word problem that leads to the equation $940 = 40(x - 12) + 20$. Then, solve that equation, and answer your word problem in words.

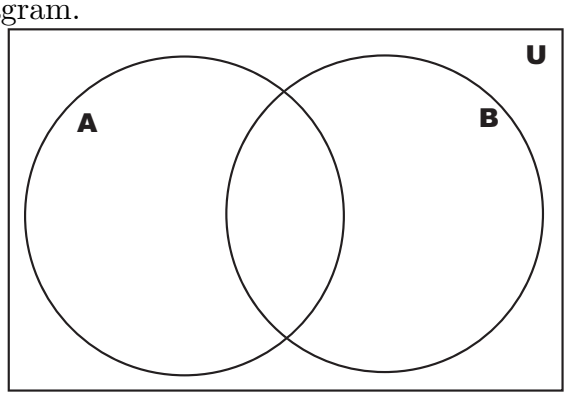
12. Explain why it's impossible to divide a whole number (that is, any of the numbers $0, 1, 2, 3, \dots$) by zero.

13. A woman earned $\$45,000$ the first year she worked. If she received a raise of $\$100,000$ at the end of each year, how much was she earning during her 11th year?

14. A woman earned $\$45,000$ the first year she worked. If her salary doubled at the end of each year, how much was she earning during her 11th year?

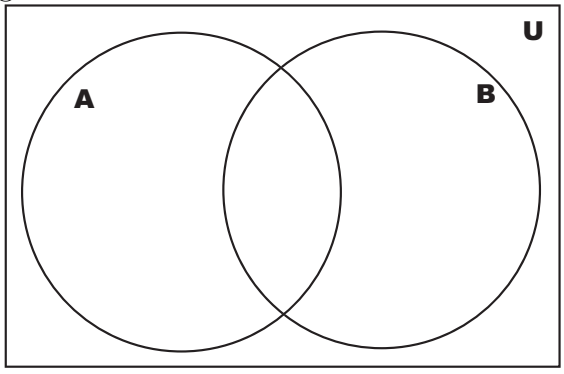
15. The *symmetric difference*, denoted $A\Delta B$, of two sets A and B is defined as follows: $A\Delta B = (A - B) \cup (B - A)$.

(a) Indicate, by shading the appropriate region, the symmetric difference $A\Delta B$ in the following Venn diagram.



(b) Why do you think it's called the symmetric difference?

(c) Indicate, by shading the appropriate region, the set $(A \cup B) \cap \overline{(A \cap B)}$ in the following Venn diagram.



(d) What can you deduce by comparing parts (a) and (c) above?

16. Suppose j, k, ℓ, and m stand for *distinct* whole numbers from 1 to 9, and that $j+k+\ell = m$.

(a) What's the smallest possible value that m can have?

(b) What digits can j, k, ℓ never be?

17. Write down the first five terms in each of the following sequences.

(a) The sequence "start with 1; multiply by 2 each time."

(b) The sequence "start with 1; then add 1; then add 2; then add 3; and so on."

(b) The sequence "start with 1; then write down successive whole numbers, skipping multiples of 3."

18. What's the moral of the previous problem?

19. Consider a sequence that starts like this: $5, a, b, c, d, 160$. Find a, b, c, and d if the sequence is:

(a) arithmetic; **(b)** geometric.

20. How many one-to-one correspondences are there between the sets $\{a, b, c, d, e, f\}$ and $\{1, 2, 3, 4, 5, 6\}$ if:

(a) a must correspond to 6 and e must correspond to 2?

(b) Each vowel must correspond to a multiple of 3?

(c) a must correspond to a number larger than 2?

(d) a must correspond to a number larger than 2, and f must correspond to a number no larger than 2?

21. Write an algebraic expression for each of the following.

(a) Height h of a box with volume V, length ℓ, and depth d

(b) Seconds s, in hours h

(c) Yards y, in inches i

(d) Hours h, in weeks w