

MATH 4510: Review for Final Exam

The final exam will cover sections 5.1-5.5, 6.1-6.5, 7.1-7.5, and 8.1-8.4. You will be permitted to use a calculator and one page (front and back) of notes on the exam. You may leave the Φ function (the cumulative distribution function of the standard normal) in your final answers.

1 Topics to Review

You should be familiar with:

1. How to compute expected values and variances for random variables with simple probability density functions.
2. How to determine the constant in a probability density function.
3. How to compute a distribution of a function of a random variable with a known distribution.
4. The uniform, normal, and exponential distributions.
5. How to compute probabilities involving multiple variables by using a joint distribution.
6. How to find marginal distributions from a joint distribution.
7. How to determine if two random variables are independent.
8. How to find a conditional distribution from a joint distribution.
9. How to find the expected value and variance of a random variable by writing it as a sum of simpler random variables.
10. How to compute expected values and variances conditionally.
11. How to compute covariance and find the correlation coefficient of two variables.
12. How to bound probabilities using the Markov and Chebyshev inequalities.
13. How to approximate probabilities using the Central Limit Theorem.

2 Sample exam

The following questions are a sample exam similar to the content and length of the actual exam. (Note that this sample exam does not cover every possible topic listed above.)

1. Let U be a uniform random variable on the interval $(1, 3)$. Let $X = \ln(U)$. Find the probability density function of X and compute the probability $P(X \leq 1)$.
2. The random variables X and Y are jointly continuous with joint probability density function $f(x, y) = cxy$ for $0 < x, y < 1$ and $f(x, y) = 0$ otherwise. Find the constant c and compute the marginal probability density functions f_X and f_Y . Find the conditional probability density function of X given that $Y = y$. Find $P(XY \leq 0.5)$.
3. A coin that lands heads with probability p is flipped n times. Let X be the number of heads and Y be the number of tails. Compute the correlation coefficient $\rho(X, Y)$.
4. A fair die is rolled 7 times. Compute the expected number of different sides that the die will land on.
5. Alex plans to complete ten tasks. She expects that she will spend an average of 12 minutes doing each task with a standard deviation of 4 minutes, independently of how long she takes on other tasks. Approximate the probability that it will take her no longer than 150 minutes to complete all of the tasks.