MATH 4510: Review for Final Exam

The final exam will consist of ten questions and will be cumulative. There will be a slight weighting towards the material since the last midterm exam. You will be permitted to use a calculator and one page (front and back) of notes on the exam. You may leave the Φ function (the cumulative distribution function of the standard normal) in your final answers.

1 Topics to Review

You should be familiar with:

1. How to count using the basic principle of counting, permutations, combinations, and multinomial coefficients.
2. What the Kolmogorov axioms say and how to verify that a function satisfies them.
3. How to compute probabilities by counting in sample spaces with equally likely outcomes.
4. How to compute probabilities using our formulas for unions, intersections, and complements.
5. How to compute conditional probabilities from the definition, by using a reduced sample space, and by using Bayes' Rule.
6. How to determine if two random variables are independent.
7. How to compute expected values and variances for random variables with simple probability mass/density functions.
8. How to determine the constant in a probability mass/density function.
9. How to compute a distribution of a function of a random variable with a known distribution.
10. The binomial, Poisson, normal, and exponential distributions.
11. How to find marginal distributions from a joint distribution.
12. How to compute probabilities involving multiple variables by using a joint distribution.
13. How to find a conditional distribution from a joint distribution.
14. How to find the expected value and variance of a random variable by writing it as a sum of simpler random variables.
15. How to compute expected values and variances conditionally.
16. How to compute covariance and find the correlation coefficient of two variables.
17. How to compute the moment generating function of a random variable and how to use it to compute the moments of the variable.
18. How to bound probabilities using the Markov and Chebyshev inequalities.
19. How to approximate probabilities using the Central Limit Theorem.

2 Sample exam

The following questions are a sample exam similar to the content and length of the actual exam. (Note that this sample exam does not cover every possible topic listed above.)

1. A die is rolled 5 times. How many different sequences of rolls are possible if:
   (a) the die never lands on 4?
   (b) the die lands on one number at least twice?
2. Alice, Bob, and Carl each attempt to solve a crossword puzzle. There is a 70% chance that Alice can solve the puzzle without making a mistake, a 60% chance that Bob can, and a 85% chance that Carl can. What is the probability that at least one of them will solve the puzzle without making a mistake?
3. In a certain area, 4% of men and 0.5% of women are colorblind. If a given person is colorblind, what is the probability that that person is a man? (You may assume that the number of men and women in the area are equal.)
4. A random variable X takes on the values -1, 3, and 5 with probabilities 0.1, 0.6, and 0.3 respectively. Find the probability mass function and cumulative distribution function of X. Compute $E(X)$ and $\text{var}(X)$.
5. Let U be a uniform random variable on the interval $(1, 3)$. Let $X = \ln(U)$. Find the probability density function of X and compute the probability $P(X \leq 1)$.
6. The random variables X and Y are jointly continuous with joint probability density function $f(x, y) = cxy$ for $0 < x, y < 1$ and $f(x, y) = 0$ otherwise. Find the constant c and compute the marginal probability density functions f_X and f_Y. Find the conditional probability density function of X given that $Y = y$. Find $P(XY \leq 0.5)$.
7. A coin that lands heads with probability p is flipped n times. Let X be the number of heads and Y be the number of tails. Compute the correlation coefficient $\rho(X, Y)$.
8. A fair die is rolled 7 times. Compute the expected number of different sides that the die will land on.
9. In a good year, the number of winter storms is Poisson distributed with parameter 3. In a bad year, the number of winter storms is Poisson distributed with parameter 5. If the next year will be a good year with probability 0.4 or a bad year with probability 0.6, find the expected value and the variance of the number of storms that will occur.
10. An exam will consist of ten questions. A student expects that she will spend an average of 12 minutes doing each question with a standard deviation of 4 minutes, independently of how long she takes on other questions. If she has 150 minutes to do the exam, approximate the probability that she will finish the exam in the allotted amount of time.