Section 2.1

8. $x_1 = -20y_1 + 7y_2$, $x_2 = 3y_1 - y_2$.

12. The matrix is invertible, with inverse $\begin{bmatrix} 1 & -k \\ 0 & 1 \end{bmatrix}$.

Section 2.2

4. This is a 45° clockwise rotation with a scaling factor of $\sqrt{2}$. See Thm. 2.2.4.

6. $\begin{bmatrix} 10/9 \\ 5/9 \\ 10/9 \end{bmatrix}$.

Section 2.3

36. $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$,

    $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$,

    $A^4 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$.

    Continuing the pattern, $A^{1001} = \begin{bmatrix} 1 & 1001 \\ 0 & 1 \end{bmatrix}$. These matrices represent horizontal shears along the x-axis. The shear strength increases linearly with each application of the matrix A.

Section 2.4

32. Use Thm. 2.4.9. We need to have

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Equating entries, we have $b = c = 0$ and $a = d$. Since $ad - bc = 1$, it must be $a = d = \pm 1$, so the only two matrices A that satisfy this requirement are I_2 and $-I_2$. (Verify that these two satisfy the given conditions.)

42. By swapping rows appropriately, we can row-reduce any permutation matrix to the identity matrix, so all permutation matrices are invertible. The inverse is also a permutation matrix, since all steps in row-reducing $\begin{bmatrix} A & | & I_n \end{bmatrix}$ are row swaps.

67. False; $(A+B)^2 = A^2 + AB + BA + B^2$. The formula in the book holds *only* if the matrices A and B commute.

69. False; one counterexample is $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

75. True; use Theorem 2.4.7 and the fact that the inverse of the inverse is the matrix itself (by Theorem 2.4.6).