Section 2.3, part 2

27. We want to show $(A+B)C = AC + BC$ (for matrices of the appropriate sizes for this to make sense). We'll use the same strategy that (most of) you used to show $A(C+D) = AC + AD$, but work with rows instead of columns. It's possible to do it without explicitly mentioning all of the theorems below, but most techniques to do so will use these ideas at least implicitly. Since most of the proofs needed below are analogous to the column versions, I've just sketched them briefly. We'll focus on column vectors in this class, so you don't need to learn/memorize these results if you don't want to.

First, we need a theorem telling us how to multiply matrices in terms of rows:

Theorem 1. *Let A be an $n \times m$ matrix with row vectors $\vec{w_1}, \ldots, \vec{w_n}$. Then the product AB is the matrix with row vectors $\vec{w_1}B, \ldots, \vec{w_n}B$. That is,*

$$\begin{bmatrix} \vec{w_1} \\ \vdots \\ \vec{w_n} \end{bmatrix} B = \begin{bmatrix} \vec{w_1}B \\ \vdots \\ \vec{w_n}B \end{bmatrix}.$$

(Note that these products are defined since the $\vec{w_j}$ are *row* vectors.)

Proof. Either follow the same strategy as the proof of Theorem 2.3.2 (slightly tricky, as you need to define row vector equivalents of $\vec{e_i}$) or use Theorem 2.3.4 to show that every entry of the matrices are equal. □

Next, we need distributivity for row vectors:

Theorem 2. *If A is an $n \times m$ matrix and $\vec{x}$ and $\vec{y}$ are row vectors in $\mathbb{R}^n$ and k is a scalar, then:*

a) $(\vec{x} + \vec{y})A = \vec{x}A + \vec{y}A$.
b) $(k\vec{x})A = k(\vec{x}A)$.

(In fact, we only need part (a) of this.)

Proof. Use the same strategy as in the proof of Theorem 1.3.10. □

Then, if we write the rows of A as $\vec{a_j}$ and the rows of B as $\vec{b_j}$,
(jth row of $(A+B)C$) = (jth row of $(A+B)$)C = $(\vec{a_j} + \vec{b_j})C = \vec{a_j}C + \vec{b_j}C$ = (jth row of AC) + (jth row of BC) = (jth row of $AC + BC$).