Answers are provided for all even numbered problems and for some odd numbered problems. If you have a question about a problem that isn't included below, feel free to ask me. If you think you've spotted an error, please let me know.

Section 1.1

12. If we subtract twice the first equation from the second, the second equation becomes $0 = 0$, so we're left with $x - 2y = 3$. If we let $y = t$, then $x = 2t + 3$.

22. We calculate $x'(t) = a\cos t - b\sin t$ and $x''(t) = -a\sin t - b\cos t$. Substituting these into the differential equation, we have

$$(-a\sin t - b\cos t) - (a\cos t - b\sin t) - (a\sin t + b\cos t) = \cos t,$$

or

$$(-2a + b)\sin t + (-a - 2b)\cos t = \cos t.$$

Setting the coefficients of $\sin t$ and $\cos t$ equal gives us the equations $-2a+b = 0$ and $-a-2b = 1$. Solving, we have $a = -\frac{1}{5}$ and $b = -\frac{2}{5}$, so $x(t) = \frac{-\sin(t)-2\cos(t)}{5}$. Note that we haven't shown that this is the only solution (as there could conceivably be some weird way to add a multiple of $\cos t$ and a multiple of $\sin t$ to get $\cos t$), but we'll see in chapter 4 that this is in fact the only solution.

24. If we let w be the speed of the water in km/min and b be the speed of the boat in km/min, then $(b + w) \cdot 20 = 8$ and $(b - w) \cdot 40 = 8$ (speed multiplied by time equals distance). Solving for w and b, we find that the boat is moving at 0.3 km/min and the water is moving at 0.1 km/min. (Answers in other units are also acceptable.)

26. Using basic row operations, we come to

$$\begin{array}{ccccccc} x & - & & & 3z & = & 1 \\ & & y & + & 2z & = & 1 \\ & & & & (k^2-4)z & = & k-2. \end{array}$$

If $k^2 \neq 4$, we can finish row reducing and find a unique solution, $x = 1 + \frac{3}{k+2}, y = 1 - \frac{2}{k+2}, z = \frac{1}{k+2}$. If $k = 2$, the last equation becomes $0 = 0$ and we find infinitely many solutions of the form $x = 1 + 3t, y = 1 - 2t, z = t$. If $k = -2$, the last equation becomes $0 = -4$ and so there are no solutions.

30. In order to come up with equations, think what it means for the points to lie on the graph. For example, $(1, p)$ lies on the graph, so $f(1) = p$. That is, $a+b(1)+c(1^2) = p$, or $a+b+c = p$. Similarly, $a + 2b + 4c = q$ and $a + 3b + 9c = r$. Row reducing, we find $a = 3p - 3q + 4, b = -\frac{5}{2}p+4q-\frac{3}{2}r$, and $c = \frac{1}{2}p-q+\frac{1}{2}r$, so that $f(t) = (3p-3q+4)+(-\frac{5}{2}p+4q-\frac{3}{2}r)t+(\frac{1}{2}p-q+\frac{1}{2}r)t^2$.

So, yes: such a polynomial exists for all p, q, and r. In fact, you can replace the x-coordinates by a,b, and c, and the same result will hold (as long as none of a,b, and c are equal).

As an aside, when will this give us a line? Note that the condition $\frac{1}{2}p - q + \frac{1}{2}r = 0$ is equivalent to $\frac{q-p}{2-1} = \frac{r-q}{3-2}$, which means that the slope between $(1, p)$ and $(2, q)$ is equal to the slope between $(2, q)$ and $(3, r)$. In other words, the only polynomial of degree ≤ 2 with three points in a straight line is a line.

Section 1.2

4. $x = 2$, $y = -1$.

18. (b) and (d) are in rref. (a) isn't since the third column contains two leading ones. (c) isn't since the third row contains a leading one, but the second row does not.

33. Let $f(t) = a + bt + ct^2 + dt^3$ and proceed as in 1.1 #30.

34. Let $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Then $\vec{v} \cdot \vec{x} = x + 3y - z$. Thus, the vector $\vec{x}$ will be perpendicular to the vector $\vec{v}$ if and only if $x + 3y - z = 0$. That is, if $\vec{x} = \begin{bmatrix} -3s + t \\ s \\ t \end{bmatrix}$ for some scalars s and t.

    Geometrically, the set of all such vectors forms the plane with normal vector $\vec{v}$ which passes through the origin. (If you took Calc III at CU in a previous semester, compare this to how the equation for a plane was first derived, on p. 831 in Section 12.6. If you're taking Calc III now, note that we've switched to a different book.)