Calculus II, Worksheet 2
Name:

Please answer the following questions in the spaces provided, or on your own paper. You may use your textbook, but do not consult any other sources or with each other. This worksheet is due on **July 30th**. As you have plenty of time for this, you will not receive credit for illegible or excessively disorganized work.

1. (20 pts) It is often difficult to find exact values for series, but with sufficient ingenuity, methods can be found. This exercise outlines one way to show that $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln 2$.

   (a) Define the sequence $a_n = \sum_{k=1}^{n} \frac{1}{k} - \ln n$. Use the Bounded Monotone Convergence Theorem to show that $\gamma = \lim_{n\to\infty} a_n$ exists. You don't need to find its value, but it so happens that $\gamma \approx 0.57722$.
   (*Hint:* Note $\ln n = \int_1^n \frac{dx}{x}$. Interpret a_n geometrically, similarly to the picture in #30 of Section 10.2)

   (b) Define $s_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}$. Define $h_n = \sum_{k=1}^{n} \frac{1}{k}$. Show that $h_n = h_{2n} - s_{2n}$.
   (*Hint:* Start with the right-hand side and simplify to the left-hand side.)

(c) Note that algebraic manipulation of (b) gives

$$s_{2n} = (h_{2n} - \ln(2n)) - (h_n - \ln n) + \ln(2n) - \ln n.$$

Using the result from (a), show that $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln 2.$

(*Hint:* What is $\lim_{n\to\infty} s_{2n}$?)