Calculus II, Worksheet 1
**Name:**

Please answer the following questions in the spaces provided, or on your own paper. You may use your textbook, but do not consult any other sources or with each other. This worksheet is due on **July 21st**. As you have plenty of time for this, you will not receive credit for illegible or excessively disorganized work.

**Philosophical Introduction:** In Calculus II, we have a number of ideas (Squeezing Theorem, Comparison Test, etc.) which rely on intuition in deciding what comparisons to make. The goals of this worksheet are to: 1) build familiarity with basic series, and to 2) look at the properties of series abstractly, to help you see which details are and are not important.

1. (10 pts) Find examples of divergent series $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ with $\lim_{k\to\infty} a_k = 0$ and $\lim_{k\to\infty} b_k = 0$ such that:

   (a) $\sum_{k=1}^{\infty}(a_k + b_k)$ converges.

   (b) $\sum_{k=1}^{\infty}(a_k + b_k)$ diverges.

   (c) $\sum_{k=1}^{\infty}(a_k b_k)$ converges.

   (d) $\sum_{k=1}^{\infty}(a_k b_k)$ diverges.

2. (10 pts)

(a) Suppose that $\{a_k\}_{k=1}^{\infty}$ and $\{b_k\}_{k=1}^{\infty}$ are sequences such that $a_k > 0$ and $b_k > 0$ for all $k$ and $\lim_{k\to\infty} b_k$ converges to a limit $B > 0$.

Show that $\sum_{k=1}^{\infty} a_k$ converges if and only if $\sum_{k=1}^{\infty} a_k b_k$ converges.

(b) Suppose that $\{a_k\}_{k=1}^{\infty}$ is a sequence such that $0 < a_k < 1$ for all $k$ and that the series $\sum_{k=1}^{\infty} a_k$ converges. Determine whether $\sum_{k=1}^{\infty} a_k^2$ converges.

(c) Suppose that $\{a_k\}_{k=1}^{\infty}$ is a sequence bounded below by $M_1 > 0$ and bounded above by $M_2$. (Note that we do not require that $\{a_k\}_{k=1}^{\infty}$ converges!)

Compute $\lim_{k\to\infty} \frac{a_k}{k}$ and determine for what values of $p$ the series $\sum_{k=1}^{\infty} \frac{a_k}{k^p}$ converges.