1. (20) Find the general solution to the simultaneous congruences

$$x \equiv 4 \pmod{11},$$
$$x \equiv 3 \pmod{16}.$$

2. (20) Let $N = 2^{35} - 1$. Show that N is not prime by finding a factor c of N other than 1 and N, justifying your answer briefly. (Hint: you do not need a calculator. For 5 bonus points: find three factors of N other than 1, N, and the factor c already found.)

3. (20) Consider the three equations

$$x^2 - 13y^2 = 7, \quad x^2 - 13y^2 = 29, \quad x^2 - 13y^2 = 47.$$

Prove that two of these equations have no solutions in integers x and y, and find a solution in integers to the third. (Hint: try to solve the equations modulo 13.)

4. (20) The number 800 factorizes as $2^5 \times 5^2$.

(i) How many factors does 800 have, including 1 and itself? (You do not need to list the factors, and there are no points for doing so.)

(ii) Evaluate $\phi(800)$, where ϕ is Euler's ϕ-function.

(iii) Find the prime factorization of $\sigma(800)$ (the sum of the divisors of 800).

5. (20) True or False. Mark with a "T" or an "F," and provide a brief explanation (a couple of lines), for each part.

(i) ______________The number 99 is a perfect number.

(ii) ______________If $n > 1$ is odd and $2^{n-1} \not\equiv 1 (\text{mod } n)$ then n is not prime.

(iii) ______________There is a unique primitive root modulo 19.

(iv) _______________ There exists a prime p with a unique primitive root modulo p.

(v) _______________ There are infinitely many primes congruent to 9 modulo 15.

(vi) _______________ If k, b, and m are natural numbers then it is possible for the congruence

$$x^k \equiv b \pmod{m}$$

not to have any solutions.

Name: ______________________________

University of Colorado

Mathematics 3110: Practice Second In-Class Exam

March 13, 2026

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	