1. (20) Show that $x^2 - 13y^2 = 2$ has no solutions in integers x and y. Hint: mod 4.

2. (20) Use the Euclidean Algorithm to find $\gcd(15343, 54203)$.

3. (20) Find a pair (x, y) of integers solving

$$15343x + 54203y = \gcd(15343, 54203).$$

(You don't need to find all solutions, just one pair (x, y).)

4. (20) For each of the following congruences, either describe all solutions in integers x, or explain why there are no solutions.

(i) $12x \equiv 6 \pmod{60}$

(ii) $6x \equiv 12 \pmod{60}$

(iii) $6x \equiv 4 \pmod{8}$

5. (20) True or False. Mark with a "T" or an "F," and provide a brief explanation (a couple of lines), for each part.

(i) ______________ If a is even and b is odd then $\gcd(a, b) = 1$.

(ii) ______________ If a, b and c are natural numbers such that $a < b < c$ and $a^2 + b^2 = c^2$, then a is odd.

(iii) ______________ The equation $60x + 85y = 5$ has infinitely many solutions in integers x and y.

(iv) _______________If a and b are natural numbers with $\gcd(a, b) > 1$, then there exists a prime number p such that $p|a$ and $p|b$.

(v) _______________If a and b are integers with $2a \equiv 2b \pmod 7$ then we must have $a \equiv b \pmod 7$.

(vi) _______________If a and b are integers with $2a \equiv 2b \pmod{10}$ then we must have $a \equiv b \pmod{10}$.

Name: ______________________________

University of Colorado

Mathematics 3110: First In-Class Exam

February 11, 2004

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	