1. (20) Let A be the 3-element set $\{\text{Rock}, \text{Paper}, \text{Scissors}\}$, and let R be the relation on A given by

$$R = \{(\text{Rock}, \text{Scissors}),\ (\text{Scissors}, \text{Paper}),\ (\text{Paper}, \text{Rock})\}.$$

(i) Determine, with justification, whether or not R satisfies the properties of being (a) reflexive; (b) irreflexive; (c) symmetric; (d) antisymmetric and/or (e) transitive.

(ii) Is R an equivalence relation? Why or why not?

(iii) Write down the inverse relation, R^{-1}, on A.

2. (40) Let $B = \{1, 2, 3\}$ be a set with three elements.

(i) Find the total number of possible relations on B.

(ii) List all the possible partitions of B. (You may leave your answer in the form of pictures if you like.)

(iii) List all the possible equivalence relations on B. For example, one of them is the equality relation,

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}.$$

(iv) Using any method you like, evaluate $\left(\!\binom{3}{3}\!\right)$ and simplify your answer as much as possible.

(v) List all the 3-element multisets on B.

3. (20) Let x and y be real numbers. Prove that if x is rational and y is irrational, then $x + y$ is irrational.

In other words, prove that

$$\forall x, y \in \mathbb{R},\ (x \in \mathbb{Q}) \wedge (y \notin \mathbb{Q}) \quad \Longrightarrow \quad x + y \notin \mathbb{Q}.$$

[Proof by contradiction will work. You may use without proof the fact that the sum or the difference of two rational numbers is rational.]

4. (20) Prove by the method of least counterexample that for every natural number $n \geq 0$, $n^3 - n$ is a multiple of 3.

[In other words, prove that $\forall n \in \mathbb{N}, 3 \mid (n^3 - n)$.]

Name: _____

University of Colorado

Mathematics 2001: Second In-Class Exam

October 30, 2019

Problem	Points	Score
1	20	
2	40	
3	20	
4	20	
Total	100	