1. (25)

(i) Prove that the product of two odd integers is odd. (In other words: let x and y be integers. Prove that if x is odd and y is odd, then xy is odd.)

(ii) Disprove the following statement: "if $x > 0$ is an integer, then $x^2 + x + 1$ is prime".

2. (15)

(i) Construct the truth table for $(\neg P) \to (P \to Q)$.

(ii) Determine whether $(\neg P) \to (P \to Q)$ is a tautology, a contradiction, or neither.

3. (15) A four-element list is made from the letters $\{A, B, C, D\}$, where repetition is allowed. An example of such a list is (C, A, D, A).

(i) How many such lists are possible? (You may leave your answers to this question as formulas.)

(ii) How many such lists are possible in which the letter A never appears?

(iii) How many such lists are possible in which the letter A appears at least once?

4. (25) Recall that $\mathbb{N}$ is the set of natural numbers $\{0, 1, 2, \dots\}$, $\mathbb{Z}$ is the set of integers, and $\mathbb{Q}$ is the set of rational numbers. Let P be the statement

$$\forall x \in \mathbb{N},\ \forall q \in \mathbb{Q},\ (q^2 = x) \Rightarrow (q \in \mathbb{Z}).$$

(i) Write out in words the meaning of P. (You do not need to determine whether P is true or false.)

(ii) Find an equivalent version (also in symbols) of the statement $\neg P$ given by

$$\neg(\forall x \in \mathbb{N},\ \forall q \in \mathbb{Q},\ (q^2 = x) \Rightarrow (q \in \mathbb{Z}))$$

that does not involve a $\neg$.

(iii) Write out, in words, the meaning of your answer to (ii).

5. (20) Let A and B be sets. Recall that the *power set* of B, denoted by $\mathcal{P}(B)$ or 2^B, is the set of all subsets of B. Prove (carefully) that if $a \in A$ and $A \in \mathcal{P}(B)$, then $a \in B$.

Name: ______________________________

University of Colorado

Mathematics 2001: First In-Class Exam

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Problem	Points	Score
1	25	
2	15	
3	15	
4	25	
5	20	
Total	100	