625-18. J. W. MOON and LEO MOSER, University of Alberta, Edmonton, Alberta, Canada. On cliques in graphs.

A complete subgraph of a graph is called a clique if it is not contained in any other complete subgraph of the graph. Erdös and Moser asked the following questions: What is the maximum number of cliques possible in a graph with n nodes and which graphs have this many cliques? These questions are answered. It is also shown that the maximum number of different sizes of cliques that can occur in a graph with n nodes is $n - \log_2 n + O(\log_2 \log_2 n)$. (Received April 12, 1965.)

625-19. W. E. KIRWAN, University of Maryland, College Park, Maryland. Extremal problems for the class of typically real functions. Preliminary report.

Let T denote the class of functions $f(z) = z + a_2 z^2 + \ldots$ which are regular in the unit circle and satisfy $\mathrm{Im}\, f(z) \cdot \mathrm{Im}\, z > 0$ for $\mathrm{Im}\, z \neq 0$. T is called the class of typically real functions. The following theorem is proved for the class T. Theorem 1. Let $|c| < 1$ and let $F(w_0, \ldots, w_{n+1})$ be analytic on $\bigcup_{f \in T} (f(c), \ldots, f^{(n)}(c), c)$. Then $\max_{f \in T} \mathrm{Re}\, F(f(c), \ldots, f^{(n)}(c), c)$ is attained for a function of the form $f(z) = z \sum_{k=0}^{n+1} s_k (1 - 2t_k z + z^2)^{-1}$ where $-1 \leq t_k \leq 1$, $s_k \geq 0$, and $\sum_{k=0}^{n+1} s_k = 1$. Using Theorem 1 the following two results are obtained. Theorem 2. Let $f(z) \in T$. Then $f(z)$ maps $|z| < \sqrt{2} - 1$ onto a domain starlike with respect to the origin. Theorem 3. Let $f(z) \in T$. Then $f(z)$ is univalent in $|z| < \sqrt{2} - 1$. (Received April 14, 1965.)

625-20. T. W. HUNGERFORD, University of Washington, Seattle, Washington 98105. A description by generators and relations of the derived functors of the n-fold tensor product.

If R is a ring (with unit) and $A^1_R, {}_RA^2_R, \ldots, {}_RA^{n-1}_R, {}_RA^n$ are R-(bi)modules, then $\mathrm{Mult}_i^{R,n}(A^1, \ldots, A^n)$ is defined to be the ith left derived functor of the n-fold tensor product $A^1 \otimes \ldots \otimes A^n$ ($\otimes = \otimes_R$); i.e., $\mathrm{Mult}_i^{R,n}(A^1, \ldots, A^n) = H_i(K^1 \otimes \ldots \otimes K^n)$ where each K^r is a projective resolution of A^r. A description of $\mathrm{Mult}_i^{R,n}(A^1, \ldots, A^n)$ is given in terms of generators and relations, analogous to that given in MacLane: Homology for the case n = 2 [and $\mathrm{Mult}_1 = \mathrm{Tor}_1^R(A^1, A^2)$]. (Received April 15, 1965.)

625-21. J. D. MONK, University of Colorado, Boulder, Colorado 80304 and F. M. SIOSON, University of Florida, Gainesville, Florida 32603. m-semigroups, semigroups, and function representations.

An m-semigroup $\langle S, (\;) \rangle$ is an algebraic structure with one m-ary operation satisfying the m-associative law $((x_1 x_2 \ldots x_m) x_{m+1} \ldots x_{2m-1}) = (x_1 x_2 \ldots x_i (x_{i+1} x_{i+2} \ldots x_{i+m}) x_{i+m+1} \ldots x_{2m-1})$ for all $i < m$ and $x_1, x_2, \ldots, x_{2m-1} \in S$. An m-semigroup $\langle S, (\;) \rangle$ is a subreduct of an ordinary or 2-semigroup $\langle A, \cdot \rangle$ iff A contains a subset S' such that $\langle S', (\;)' \rangle$ is an m-semigroup isomorphic to $\langle S, () \rangle$ under the operation defined by $(x_1 x_2 \ldots x_m) = x_1 \cdot x_2 \cdot \ldots \cdot x_m$ for all $x_1, x_2, \ldots, x_m \in S'$. A disjoint m-semigroup of m-adic transformations is an m-semigroup of functions $f: \bigcup_{i=1}^{m-1} X_i \to \bigcup_{i=1}^{m-1} X_i$ such that $f(X_i) \subseteq X_{\sigma(i)}$ (σ being the cyclic permutation $(1\ 2 \ldots m-1)$) under the operation of composition of any m such functions, and where $X_1, X_2, \ldots, X_{m-1}$ are any m - 1 pairwise disjoint sets. Theorem 1. Any m-semigroup $\langle S, (\;) \rangle$ is the subreduct of an ordinary semigroup $\langle A, \cdot \rangle$ such that S generate A and if $\langle S, () \rangle$ is a subreduct of any other semigroup $\langle B, \cdot \rangle$, then a homomorphism exists from A into B which is the identity on S. Theorem 2. Every m-semigroup is isomorphic to a disjoint m-semigroup of m-adic transformations. Theorem 3. An m-semigroup is isomorphic to a disjoint m-semigroup of one-to-one m-adic transformations iff it is left-cancellative. The Post Coset Theorem is a corollary of the above result. (Received April 15, 1965.)

625-22. J. S. WHITE, General Motors Corporation, Warren, Michigan. Series expansions for order statistics.

Let $X_1 \leqq \ldots \leqq X_n$ be the order statistics of a random sample of size n from a population with continuous distribution function $F(x)$; then $U_i = F(X_i)$ is distributed as the ith order statistic from a uniform (0,1) distribution. Let $G(u)$ be the inverse of $F(x)$ (i.e., $u = F(x)$ implies $G(u) = x$); then the Taylor expansion of $G(u)$ about $u = p$ is $x = G(u) = \Sigma(u - p)^k d^k G(p)/dp^k/k!$. If we set $X_i = x, p = i/(n+1) = E(U_i)$, we obtain an expansion for X_i in terms of $(U_i - p)^k$ and hence an expansion for $E(X_i)$ in terms of the central moments of U_i. Expansions for higher moments of X_i and cross moments $E(X_i^r X_j^o)$ may be obtained by multiplying appropriate series. These series expansions are generally asymptotic expansions and may not give sufficient accuracy for small n. Examples for the normal, half normal, and log Weibull distributions are given. (Received April 22, 1965.)

625-23. J. E. SIMPSON, Marquette University, Milwaukee Wisconsin 53233. On dilations of operators.

Let 1 and Z be the functions $1(z) = 1$ and $Z(z) = z$ for all $z \in C$, $B^{\infty}(C)$ the Banach algebra of complex-valued bounded Borel functions on C, E a separated complete locally convex topological vector space over C with strong dual E', and L(E,E) the algebra of continuous linear mappings of E into itself with the topology of uniform convergence on bounded subsets of E. A family $\mathcal{F} = (m_{x,x'})_{x \in E, x' \in E'}$ of bounded Radon measures on C will be called spectral if there is a mapping $f \to U_f$ of $B^{\infty}(C)$ into L(E,E) which is a continuous algebra representation satisfying $\langle U_f x, x' \rangle = \int_C f \, dm_{x,x'}$ for all $f \in B^{\infty}(C)$, $x \in E$, $x' \in E'$, and $U_1 = I$. An operator $T \in L(E,E)$ is scalar if there is a spectral family such that Z is $m_{x,x'}$-integrable and $\langle Tx, x' \rangle = \int_C Z \, dm_{x,x'}$ for all $x \in E$, $x' \in E'$. Theorem. Let $(T_j)_{j=1,2,\ldots,n}$ be a finite subset of L(E,E). Then there is a separated complete space F which contains E, an idempotent element P of L(F,F) with range E, and n regular (i.e., with compact spectrum) commuting scalar operators $\widetilde{T}_j$ in L(F,F) such that $T_j x = P\widetilde{T}_j x$ for all $x \in E$, $j = 1,2,\ldots,n$. Conditions will also be given under which an element T of L(E,E) is the restriction of a scalar operator and not merely its projection. (Received April 22, 1965.)

625-24. HERBERT GROSS, Montana State College, Bozeman, Montana 59715. On Witt's theorem in the countably infinite case.

Theorem. Let ϕ be a non-degenerate, alternate form on a k-vectorspace E of denumerable dimension, k an arbitrary commutative field (of any characteristic). Let H and $\bar{H}$ be subspaces of E with $\mathrm{rad}(H) = \mathrm{rad}(H^{\perp})$, $\mathrm{rad}(\bar{H}) = \mathrm{rad}(\bar{H}^{\perp})$ satisfying the following conditions: (i) $\dim H = \dim \bar{H}$, $\dim(H/\mathrm{rad}\,H) = \dim(\bar{H}/\mathrm{rad}\,\bar{H})$, $\dim(H + H^{\perp}/H) = \dim(\bar{H} + \bar{H}^{\perp}/\bar{H})$, (ii) $\dim(H^{\perp} + H^{\perp\perp}/H^{\perp} + H) = \dim(\bar{H}^{\perp} + \bar{H}^{\perp\perp}/\bar{H}^{\perp} + \bar{H})$, $\dim((\mathrm{rad}H)^{\perp}/H^{\perp} + H^{\perp\perp}) = \dim((\mathrm{rad}\,\bar{H})^{\perp}/\bar{H}^{\perp} + \bar{H}^{\perp\perp})$. Then there exists a metric automorphism of E which maps H onto $\bar{H}$. The theorem also holds in the case of a symmetric form.