61T-166. HARRY KESTEN, The Hebrew University of Jerusalem, Israel. Some probabilistic theorems on diophantine approximations.

Let $\langle \xi \rangle$ be the (positive) distance between ξ and the integer closest to ξ. Instead of a fixed ξ a random x is chosen with a uniform distribution on $[0,1]$ and the limiting distribution ($m \longrightarrow \infty$) of $m \cdot \min_{1 \leq k \leq m} \langle kx \rangle$ is determined. At the same time the limiting distribution ($m \longrightarrow \infty$) of the smallest integer k for which $\langle kx \rangle \leqq m^{-1}$ is found. Going to higher dimensions, let $x_1, x_2, \ldots$ be independent random variables, each with a uniform distribution on $[0,1]$ and define: $N(m,\gamma,p)$ = the number of integers $k, 1 \leqq k \leqq m$, for which simultaneously $\langle kx_1 \rangle \leqq \gamma, \langle kx_2 \rangle \leqq \gamma, \ldots, \langle kx_p \rangle \leqq \gamma$. Then it is shown that for fixed $0 < \gamma < 1/2$ the distribution of $N(m,\gamma,p)$ tends to a Poisson distribution with mean λ if $p \longrightarrow \infty$, $m \longrightarrow \infty$ such that $m(2\gamma)^p \longrightarrow \lambda$. (Received May 22, 1961.)

61T-167. R. J. BUEHLER, Statistical Laboratory, Iowa State University, Ames, Iowa. New proofs and generalization of an optimum-gradient theorem.

In an optimum-gradient iteration, successive values $\phi_1, \phi_2, \ldots$, of a positive definite quadratic function $\phi = x'Ax$ satisfy $\phi_2^2 \leq \phi_1\phi_3$ for any initial vector x_1, and hence $\phi_{n+2}/\phi_{n+1} \geqq \phi_{n+1}/\phi_n$. Proof: It is easily shown that $x_2 = (I + \lambda A)x_1$ where $\lambda = - x_1'A^2x_1/x_1'A^3x_1$, and $x_3 = (I + \mu A)x_2$, whence $x_1'Ax_3 = x_1'(I + \lambda A)Ax_1 = x_2'Ax_2$. By Schwarz's inequality, $\phi_2^2 = (x_1'Ax_3)^2 \leqq (x_1'Ax_1)(x_3'Ax_3) = \phi_1\phi_3$. In a second proof, ϕ is represented in a canonical form $\sum a_i x_i^2$, and $3(\phi_1\phi_3 - \phi_2^2)/[\phi_1(\phi_2 - \phi_3)]$ is shown to equal $(\sum z_i^2 a_i)^2 \sum\sum\sum z_i^2 z_j^2 z_k^2 (a_i - a_j)^2 (a_j - a_k)^2 (a_k - a_i)^2 / [\sum\sum z_i^2 z_j^2 a_i a_j (a_i - a_j)^2]^2$ where $z_i^2 = a_i x_{1i}^2/\phi_1$. The first proof is seen to apply equally to the Hilbert space iteration considered by Temple (Proc. Roy. Soc. London Ser. A vol. 169 (1939) pp. 476-500) with A a positive Hermitian operator. In this respect the result is more general than that given by Akaike (Ann. Inst. Statist. Math. Tokyo vol. 11 (1959) pp. 1-16). (Received May 22, 1961.)

61T-168. DONALD MONK, University of California, Berkeley, California. Relation algebras and cylindric algebras.

Let $\mathfrak{A}$ be a CA_α with $\alpha > 2$ (Henkin and Tarski, Cylindric algebras, Proceedings of Symposia in Pure Mathematics, vol. 2, Amer. Math. Soc., 1961, pp. 83-113). Let $R(\mathfrak{A}) = \langle R(A), +, \cdot, -, ;, ^{\cup}, 1' \rangle$, where $R(A) = \{a: \Delta a \subseteq 2\}$, $x;y = c_2(c_1(d_{12} \cdot x) \cdot c_0(d_{02} \cdot y))$ for all $x,y \in R(A)$, $x^{\cup} = c_2(d_{12} \cdot c_1(d_{01} \cdot c_0(d_{02} \cdot x)))$ for all $x \in R(A)$, and $1' = d_{01}$. Under certain conditions, $R(\mathfrak{A})$ is a relation algebra (in the sense of Chin and Tarski, University of California Publications in Mathematics, new series, vol. 1, no. 9, pp. 341-384). Conversely, given a relation algebra $\mathfrak{A}$ we construct a CA_3 $C(\mathfrak{A})$, using an idea of Lyndon. Theorem 1. If $\mathfrak{A}$ is a relation algebra, then $C(\mathfrak{A})$ is a cylindric algebra, and $\mathfrak{A} \cong RA(\mathfrak{A})$. Also, $\mathfrak{A} \cong \mathfrak{B}$ if and only if $C(\mathfrak{A}) \cong C(\mathfrak{B})$. Theorem 2. There is a relation algebra $\mathfrak{A}$ such that for no CA_5 $\mathfrak{B}$ do we have $\mathfrak{A} \cong R(\mathfrak{B})$. (Received May 23, 1961.)

61T-169. STEFAN BERGMAN, Stanford University, Stanford, California. Bounds for functions of two complex variables in a domain with a distinguished boundary set.

Let $\mathfrak{M}$ be a domain with the "smallest maximum set" $\mathfrak{F}$ in the space of two complex variables, and let $\mathfrak{F}$ be a proper part of the boundary of $\mathfrak{M}$ (See Bergman, Über eine in gewissen Bereichen mit