CU Boulder

Math 2130

Test 2

Section 002 (Instructor Farid Aliniaeifard)

Friday, Oct 6, 2017, 10:00 - 10:40 am

NAME (print):____
(Family) (Given)

SIGNATURE:____

STUDENT NUMBER:____

Instructions:

1. Time allowed: 40 minutes.
2. NO CALCULATORS OR OTHER AIDS
3. There are 5 questions on 5 pages. Last page is blank.
4. Questions can be solved in more than one way.
5. You are expected to write clearly and carefully.

Question	**Points**	**Marks**
1	10	
2	10	
3	10	
4	10	
5	5	
Total	45	

1. (10 points) Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}.$$

Is A diagonalizable? If so write $A = PDP^{-1}$, where P is invertible and D is diagonal.

2. (10 points)

   (a) Write a formula for the determinant of an $n \times n$ matrix A.

   (b) Let $\{v_1, \ldots, v_p\}$ is a subset of a vector space W. What does it mean $W = sapn\{v_1, \ldots, v_n\}$? What does it mean $\{v_1, \ldots, v_n\}$ is linearly independent.

   (c) A subspace of a vector space W is ....

   (d) Let A be an $n \times n$ matrix. The characteristic equation of A is ...

3. (10 points) Let W be a vector space with a basis $\{v_1, v_2, v_3\}$. Define two other bases for W,

$$\mathcal{B} = \{v_1 - v_2, -v_3, v_1 + v_3\} \quad \text{and} \quad \mathcal{C} = \{v_1 + v_2, v_2 + v_3, v_1 + v_2 + v_3\}.$$

(a) Find $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$.

(b) Let $w = 2v_1 + 4v_2 + 3v_3$. Write $[w]_{\mathcal{B}}$.

4. (10 points) Let V be a vector space with a basis $\mathcal{B} = \{v_1, v_2, v_3\}$ and W a vector space with a basis $\mathcal{C} = \{w_1, w_2, w_3\}$. Let T be a function from V to W such that $T(v_1) = w_1 + w_2$, $T(v_2) = w_1 + w_2 - w_3$, and $T(v_3) = 2w_1 + 2w_2 - w_3$.

   (a) Is T a linear transformation? Justify your answer.

   (b) If T a linear transformation, write matrix T relative to the basis $\mathcal{B}$ and $\mathcal{C}$.

5. (5 points) Mark each statement True or False.

   (a) If $A = PDP^{-1}$ for an invertible matrix P and a diagonal matrix D, then P and D are unique.

   (b) If v_1 and v_2 are linearly independent eigenvectors, then they correspond to different eigenvalues.

   (c) Let T be a linear transformation. Then T is one-to-one if and only if $ker\ T = \{0\}$.

   (d) If $T : V \to W$ is an isomorphism and H is a subspace of V of dimension n, then image of H is also of dimension n.

   (e) If $W = span\{v_1, v_2, v_3, v_4, v_5\}$ and $v_1 \in span\{v_2, v_3, v_4, v_5\}$, then $dimW = 4$.

The end. Have a great weekend